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SAT Mathematical Concepts

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Complex Numbers

Theory

The square of any real number is non-negative. The number *i*. Is defined to be the solution to equation $x^2 = -1$, or we can say $i^2 = -1$.
$i = \sqrt{-1}$; $-1 = i^2$

i or iota is defined as square root of −1.

A complex number can be represented in the from $z = a + bi$, where a and b are real numbers and $I = \sqrt{-1}$. This is known as standard form of a complex number. The number *a* is called real part of *z*, and *bi* is called imaginary part of *z*.

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Power of i

$$i^1 = i \quad i^2 = -1 \quad i^3 = -i \quad i^4 = 1$$
$$i^5 = i \quad i^6 = -1 \quad i^7 = -i \quad i^8 = 1$$
$$i^9 = i \quad i^{10} = -1 \quad i^{11} = -i \quad i^{12} = 1$$

Example

Find the value of √-64 + √-16 +√-4

Solution

√-64 + √-16 +√-4

= 8i + 4i +2i

= 14i

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Arithmetic of Complex Numbers

Addition

For addition, we add real parts separately and imaginary parts separately.

For example

$Z_1 = 6 + 5i,\ Z_2 = 2 + 9i$

$Z_1 + Z_2 = 6 + 5i + 2 + 9i$

$= (6+2) + (5+9)i$

$= 8 + 14i$

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Subtraction

For subtraction, we subtract real parts separately and imaginary parts separately.

For example

$Z_1 = 6 + 5i$, $Z_2 = 2 + 9i$

$Z_1 + Z_2 = (6+5i) - (2+9i)$

$= 6 + 5i - 2 - 9i$

$= (6-2) + (5-9)i$

$= 4 - 4i$

Multiplication

Multiplication is performed simply like binomials, using the fact $i^2 = -1$.

For example

$Z_1 = 6 + 5i$, $Z_2 = 2 + 9i$

$Z_1 + Z_2 = (6+5i)(2+9i)$

$= (6)(2) + (6)(9i) + (5i)(2) + (5i)(9i)$

$= 12 + 54i + 10i + (45)i^2$

$= 12 + 64i - 45$

$= -33 + 64i$

Conjugate of a complex number

The complex number $z_1 = a - bi$ is known as complex conjugate of $z = a + bi$.

The product of z and z_1 will be

$$(a+bi)(a-bi) = a^2 - abi + abi - (b^2)i^2$$

$$= a^2 + b^2$$

A and b are real numbers , so $a^2 + b^2$ will also be a real number. The product of a complex number and its conjugate is always a real number.

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Division

Whenever a complex number is in the denominator, we multiply the denominator and numerator by the conjugate of the denominator. This makes denominator a real number, and we perform multiplication for getting the numerator.

For example

$Z_1/Z_2 = (6+5i)/(2+9i)$

$= (6+5i)(2-9i)/(2+9i)(2-9i)$

$= (6+5i)(2-9i)/2^2+9^2$

$= (12-54i+10i+45)/85 = 57-44i/85$

Geometry

Theory

SAT does not require you to remember complex formulas. Rather, one should focus on the basic concepts and master the application part of geometry.

Plane Geometry

Few basic terms and properties:

1. Acute angle: Angle less than 90°.

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2. Right angle: Angle equal to 90°.

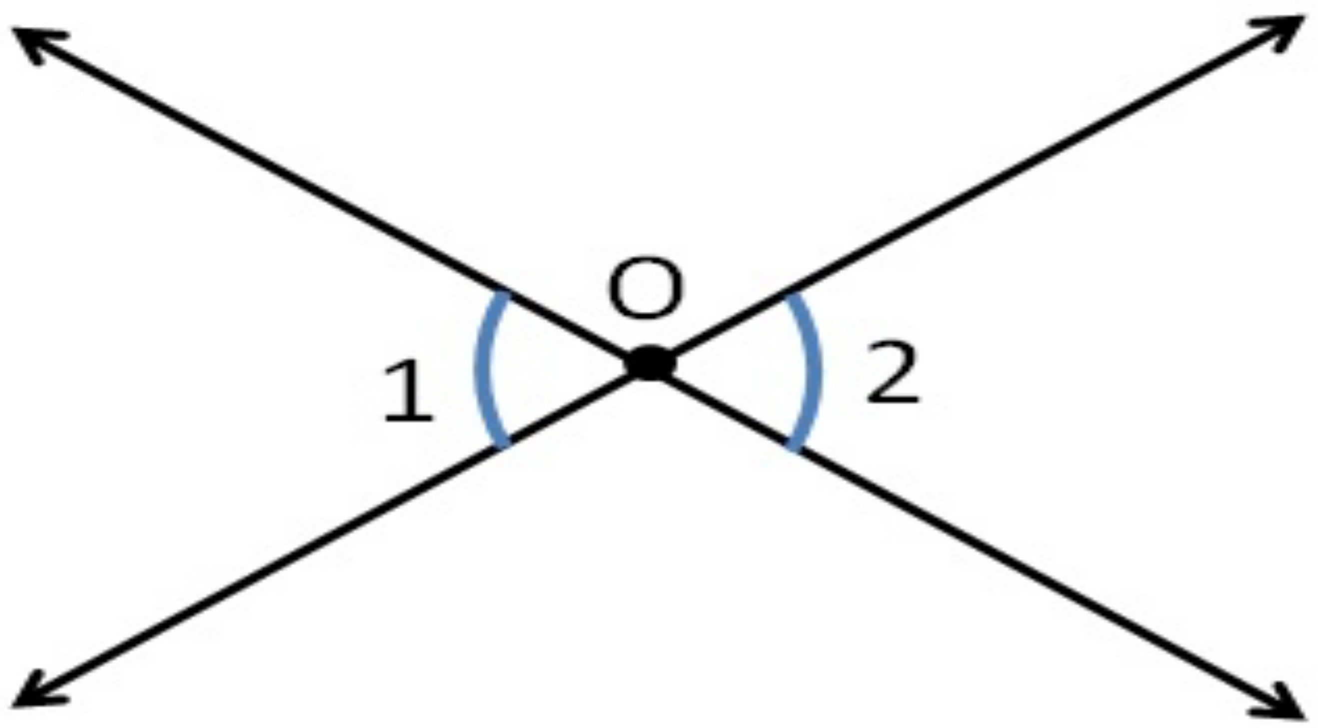
3. Obtuse angle: Angle greater than 90° and less than 180°.

4. Angle on a straight line is 180°. Angle about a point is 360°.

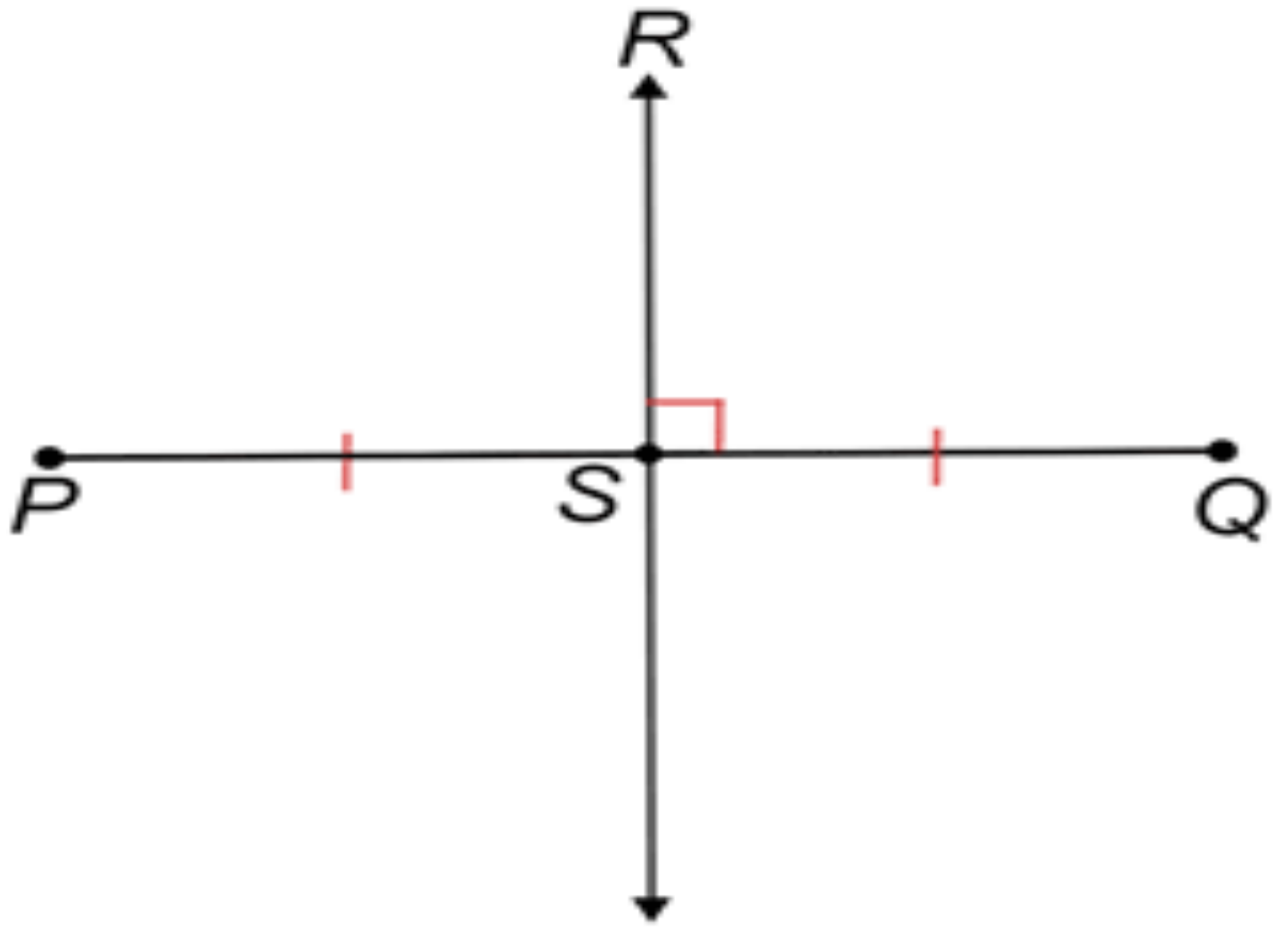
5. Supplementary angles: Two angles which add unto 180°.

6. Complementary angles: Two angles which add unto 90°.

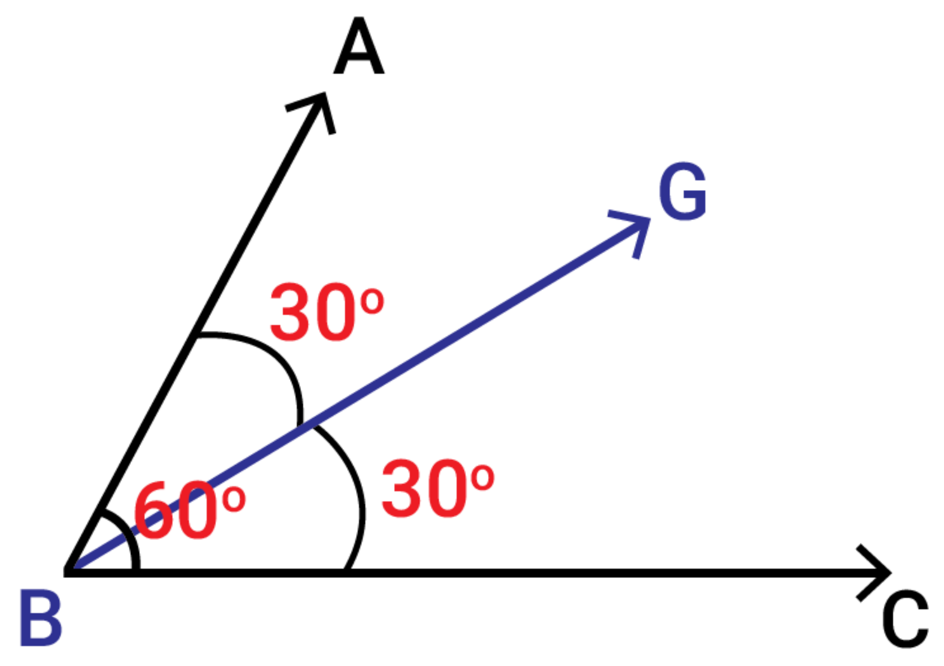
7. Vertically Opposite Angles: Two intersecting lines form 4 angles, and non-adjacent angles are called vertically opposite angles. Vertically opposite angles are equal. Here, angle b°= angle a°.



8. Perpendicular Bisector: A perpendicular bisector can be defined as a line segment which intersects another line perpendicularly and divides it into two equal parts. Two lines are said to be perpendicular to each other when they intersect in such a way that they form 90 degrees with each other.



9. Angle Bisector: A line which bisects an angle. Any point on angle bisector is equidistant from the arms of the angle.



BG - Angle bisector of 60°

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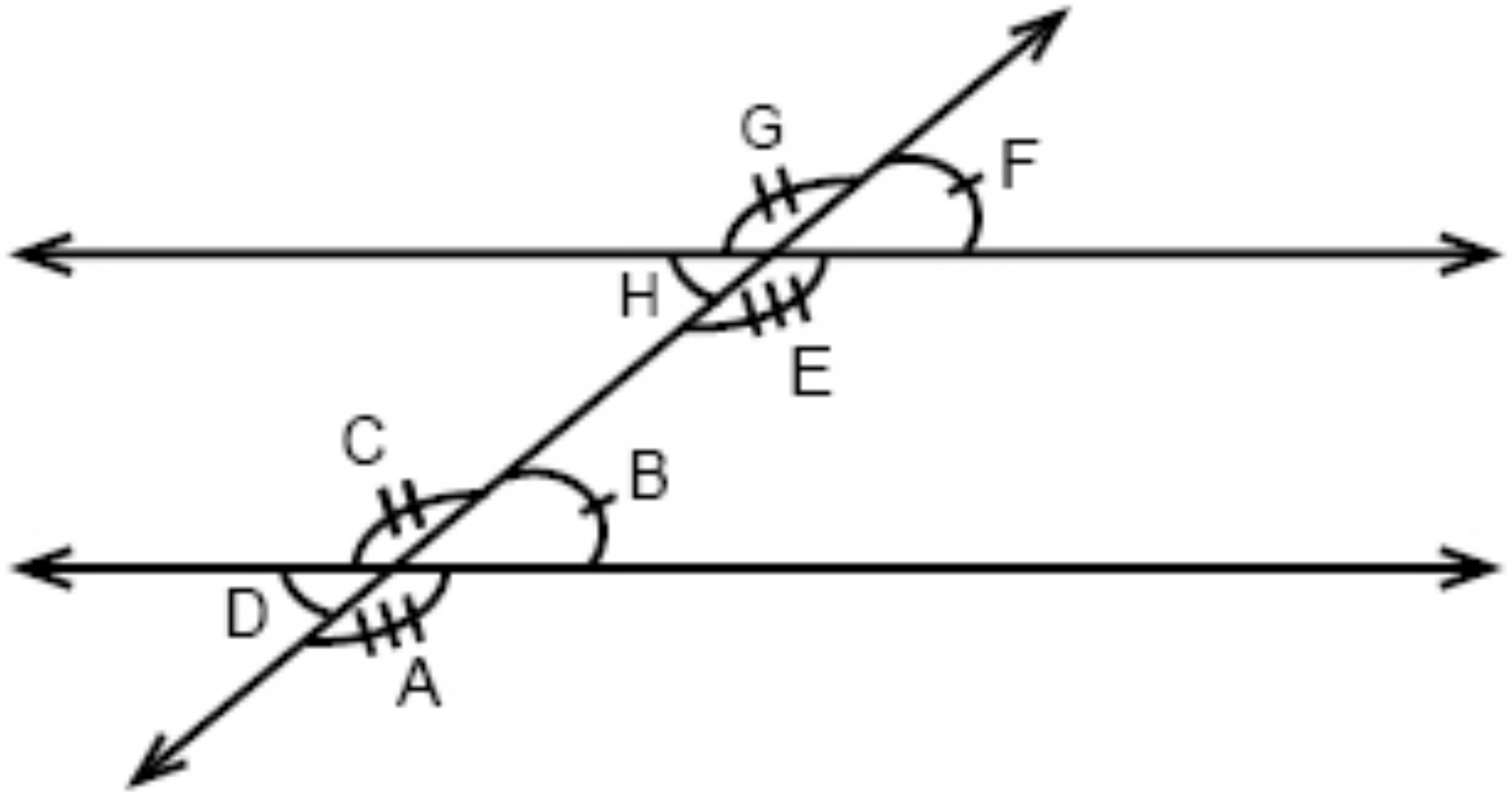
Parallel lines

Coplainar lines which do not intersect are called parallel lines (denoted by symbol ||). If a line intersects two parallel lines, then the intersecting line is called Transversal. For the resulting 8 angles, following terms are defined:



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1. Corresponding angles: Angles on the same side of transversal and same side of parallel lines. Corresponding angles are equal.
e.g. ∠A=∠C, ∠B=∠D, and so on.

2. Alternate interior angles: Interior angles on alternate side of transversal. Alternate Interior angles are equal.
e.g. ∠B=∠H and ∠C=∠E.

3. Interior angles on the same side of transversal are supplementary.

Triangles

Figure obtained by joining 3 non-collinear points is called Triangle (denoted by symbol △). Any triangle will satisfy these properties:

1. Sum of all interior angles is 180°. It can also be said as: Exterior angle is the angle supplementary to adjacent interior angle.

2. Sum of lengths of any 2 sides is always greater than the length of the third side. It can also be said as: Difference between the lengths of any 2 sides is always less than the length of the third side.

3. Greater angle has greater side opposite to it.

In △ABC, if ∠B>∠C

it implies that, AC>AB

Classification of Triangles (on the basis of sides)

1. Equilateral Triangle: an equilateral triangle is a triangle in which all three sides have the same length. All three internal angles are equal and are 60° each. It is also referred to as a regular triangle.

2. Isosceles Triangle: an isosceles triangle is a triangle that has two sides of equal length. Internal angles opposite to each side are equal.

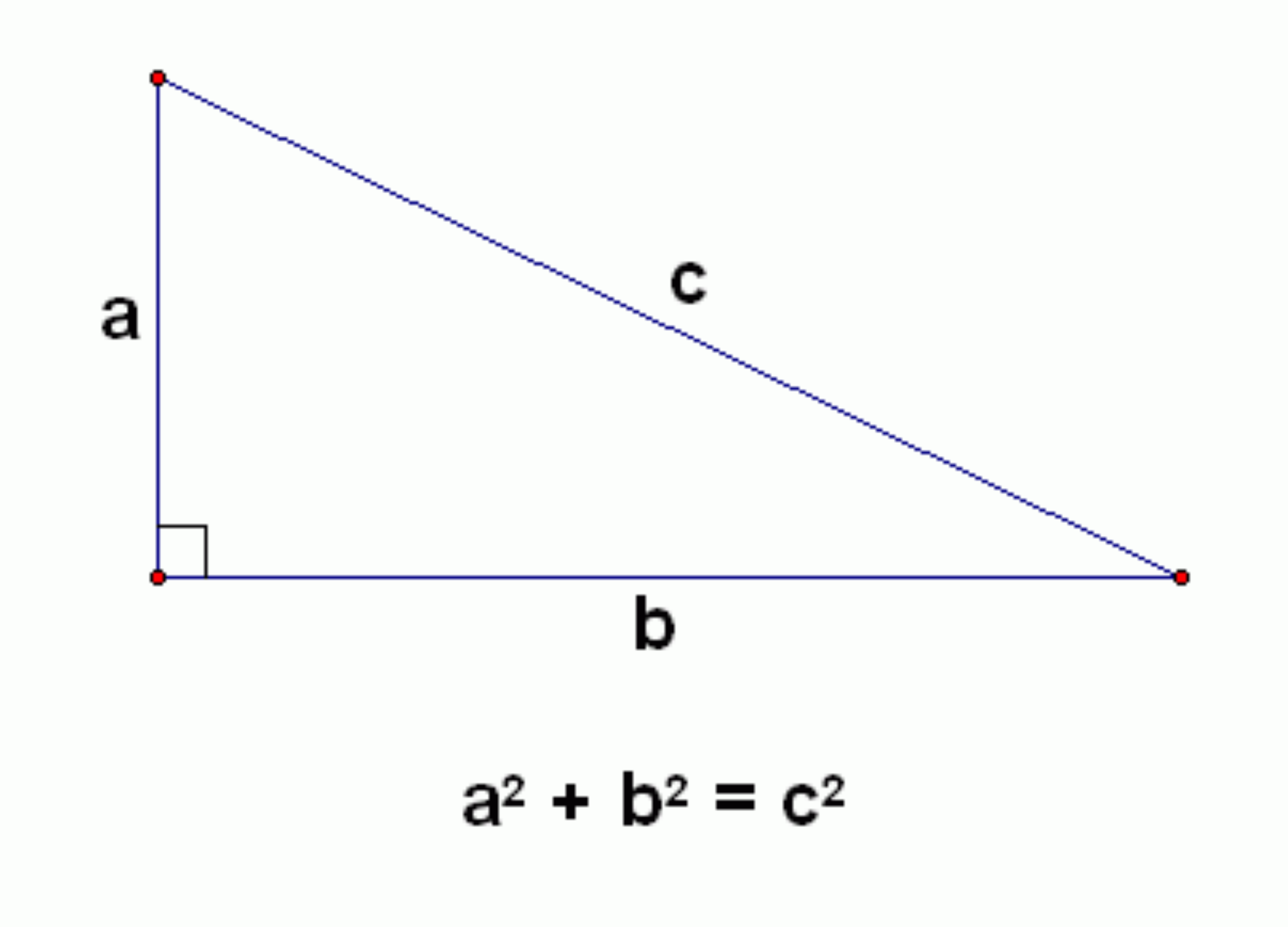
3. Scalene Triangle: Triangle in which no two sides are equal. All interior angles are also unequal.

Classification of Triangles (on the basis of angles)

4. Right Triangle: Triangle in which one angle is equal to 90°. Side opposite yo largest angle (90°) is called hypotenuse.

Pythagorean theorem

Square of the length of the largest side=sum of the squares of the lengths of the remaining 2 sides.

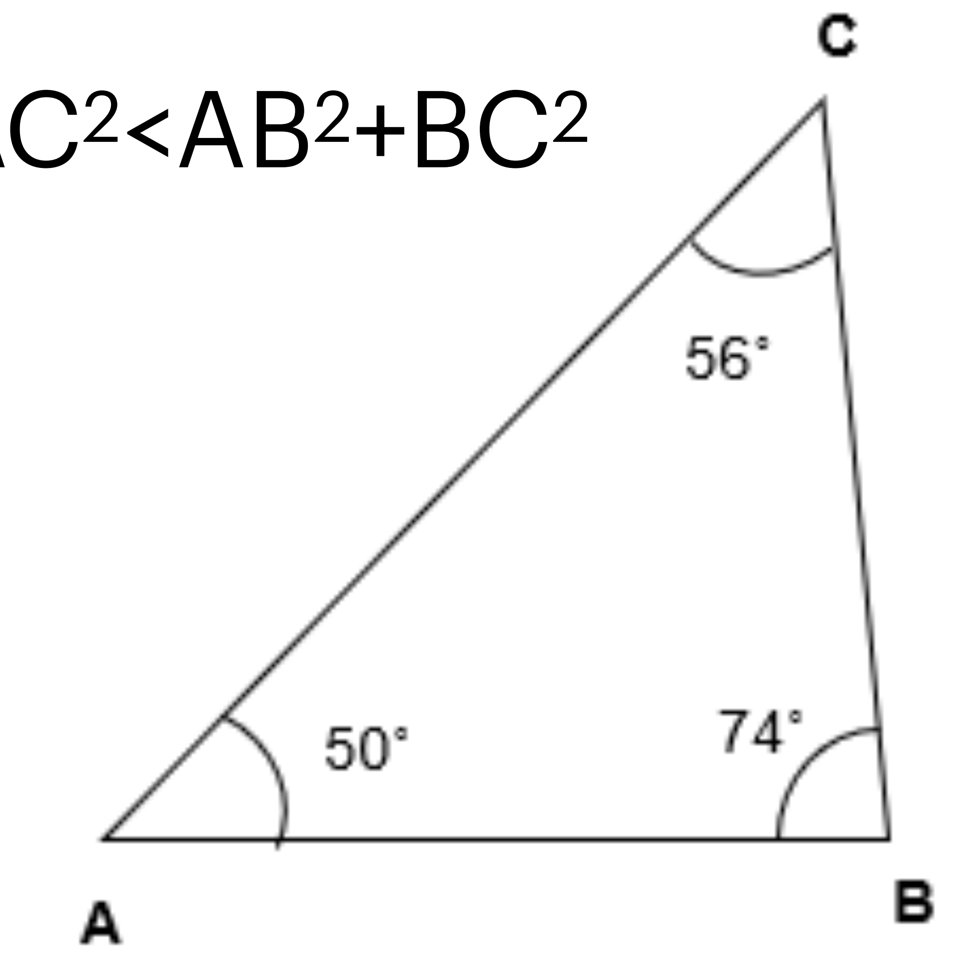


Perimeter = a + b + c

Area = 1/2 x b x h

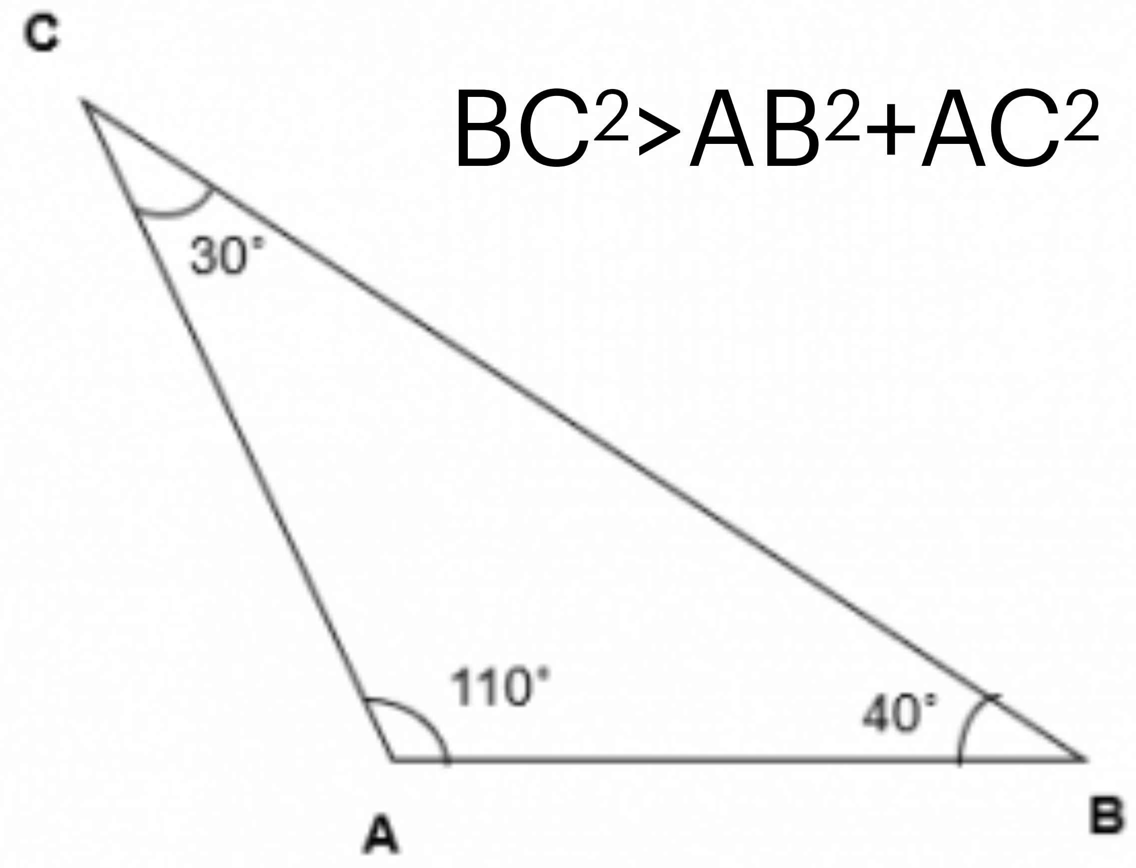
5. <u>Acute angled triangle</u>: triangle in which all angles are less than 90°.
square of the length of the largest side < sum of the square of the lengths of the remaining 2 sides.

$AC^2 < AB^2 + BC^2$



6. <u>Obtuse angled triangle</u>: Triangle in which one angle is greater than 90°.
square of the length of the largest side > sum of the square of the lengths of the remaining 2 sides

$BC^2 > AB^2 + AC^2$



Formulae

- Area of a triangle = 1/2 x base x height
- Area of an equilateral triangle = √3/4 x side^2

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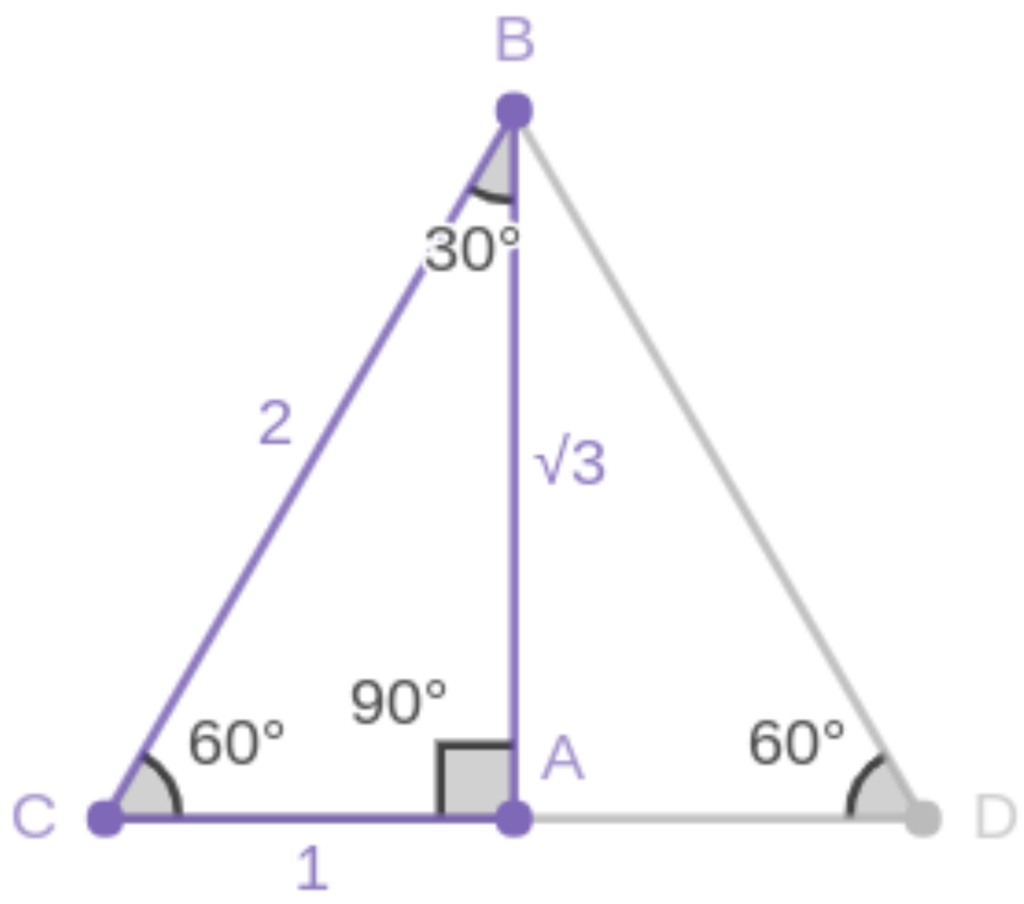
<u>Two very important right triangles</u>



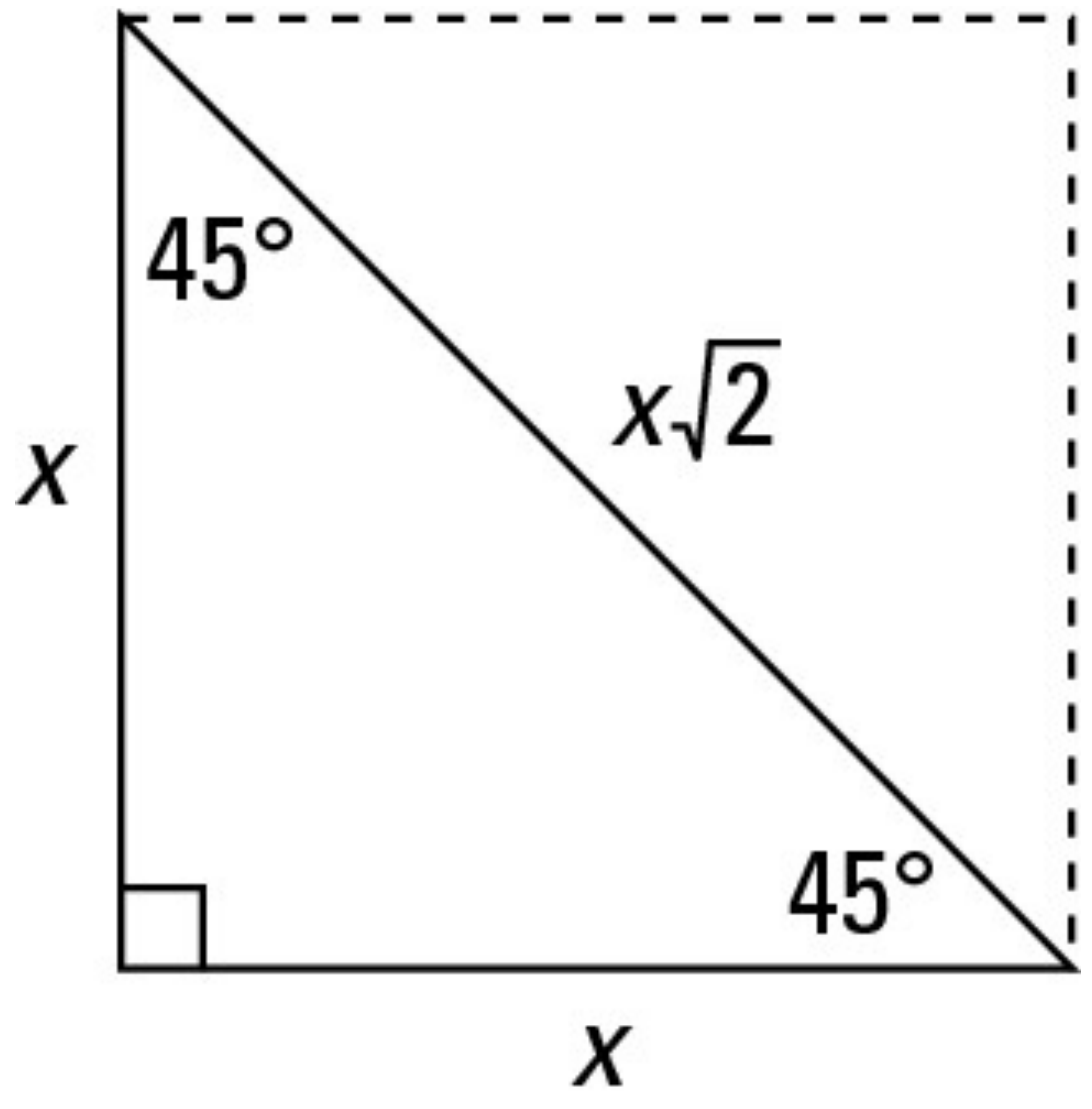
1. <u>30°-60°-90° triangle</u>: Right triangle in which two of the angles are 30° and 60° (then third angle has to be 90°). An equilateral triangle can be formed by joining two identical 30°-60°-90° triangles side by side.
Hence, if length of AC is x, then length of BC is x/2 and length of AB is √3x/2 (by Pythagorean theorem), i.e.

<u>Length of the side opposite to 30°= Half of the length of the hypotenuse</u>

<u>Length of the side opposite to 60° = √3 x (length of the side opposite to 30°)</u>

<u>Length of the sides of 30°-60°-90° triangle are in the ratio of 1:√3:2.</u>

2. <u>45°-45°-90° triangle:</u> Right triangle in which two of the angles are 45° each (then the third angle has to be 90°). Since it is an isosceles triangle, sides opposite to 45° (AB and BC) are equal. Hence, if length of AB=BC=x, then length of AC is √2x (by using Pythagorean theorem), i.e.



<u>Length of the Hypotenuse= √2 x (length of the equal sides).</u>

<u>Length of sides of 45°-45°-90° triangle are in the ratio of 1:1:√2.</u>

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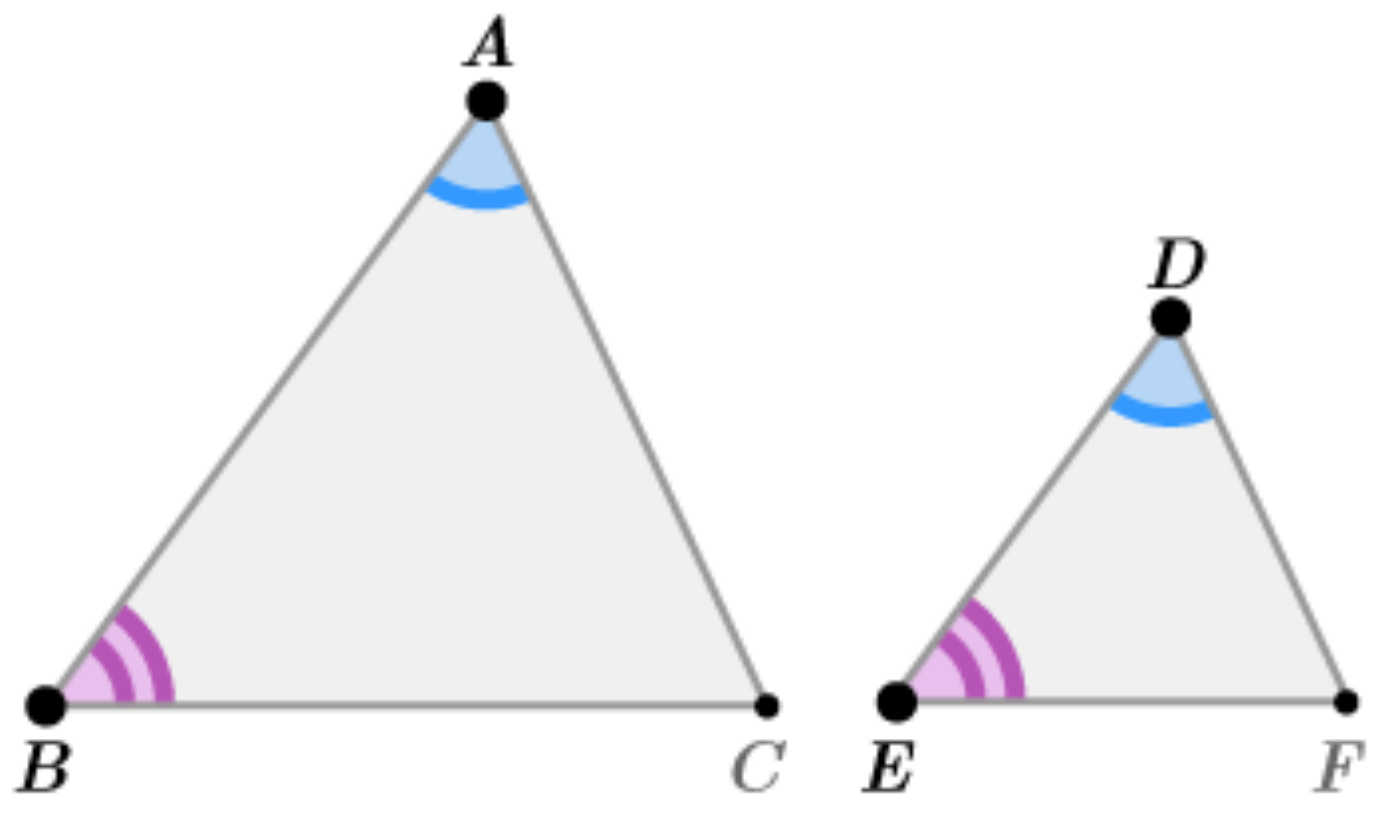
Similarity

Two triangles are said to be similar, if the interior angles of one triangle are respectively equal to interior angles of the other triangle. For e.g. consider following two triangles:



Let ∠A=∠D; ∠B=∠E; and ∠C=∠F

Therefore, △ABC is similar to △DEF. In similar triangles, every respective dimension (i.e. length) increases or decreases by the same factor.

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Quadrilaterals

Quadrilateral is a polygon with four sides and four vertices. In any quadrilateral, sum of all interior angles is 360°.

Types of quadrilateral:

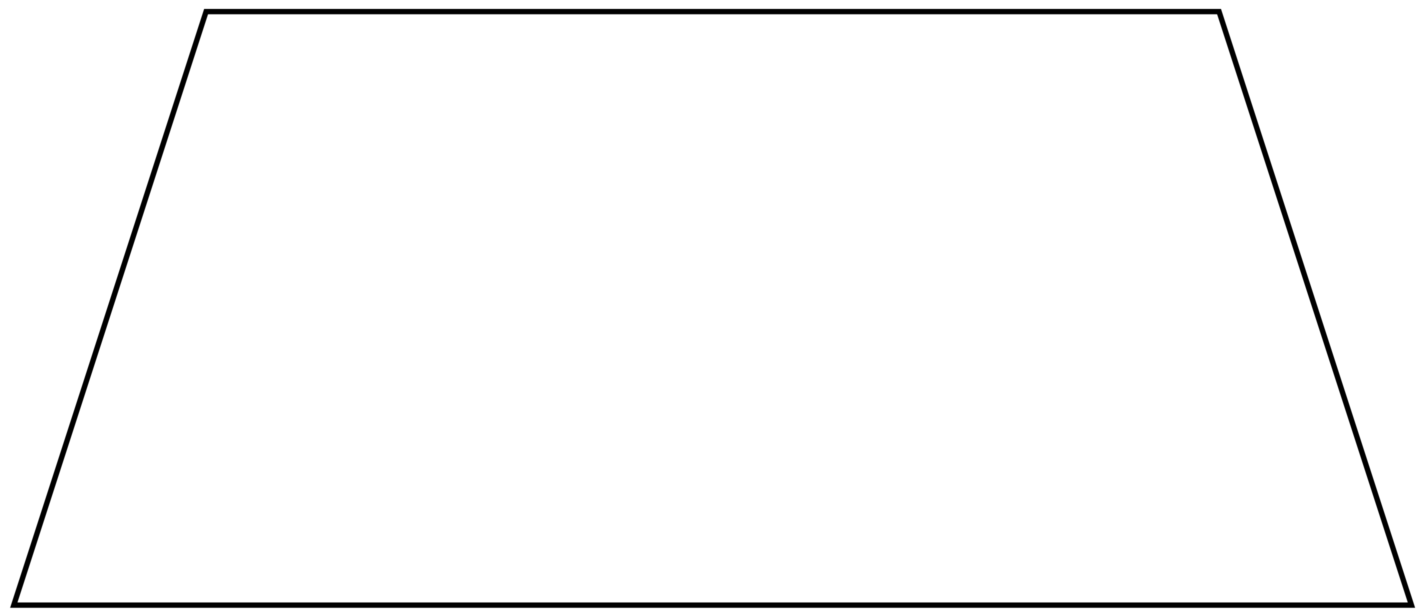
Figure	**Definition and Properties**	**Area of the figure**
 **Trapezium**	Quadrilateral with one pair of parallel sides. Parallel sides are called bases and non-parallel sides are called legs.	1/2 x (sum of parallel sides) x (distance between them)

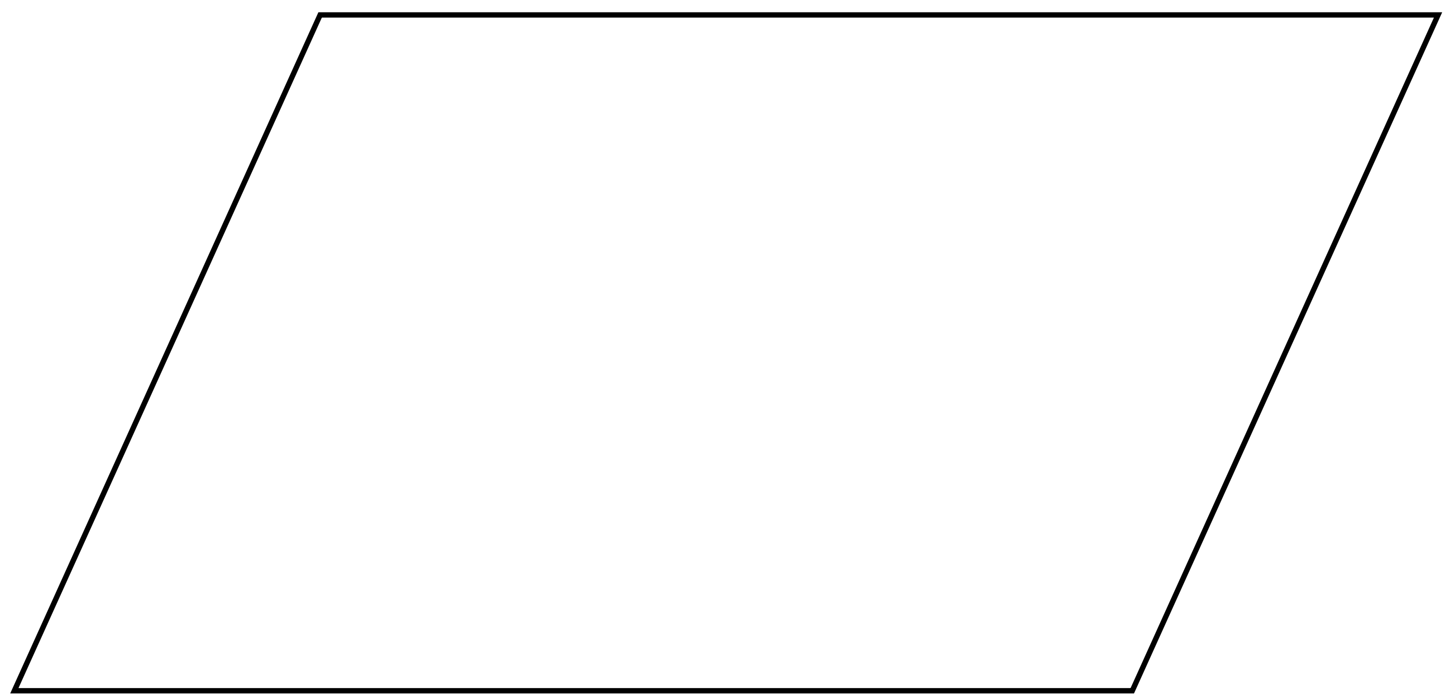
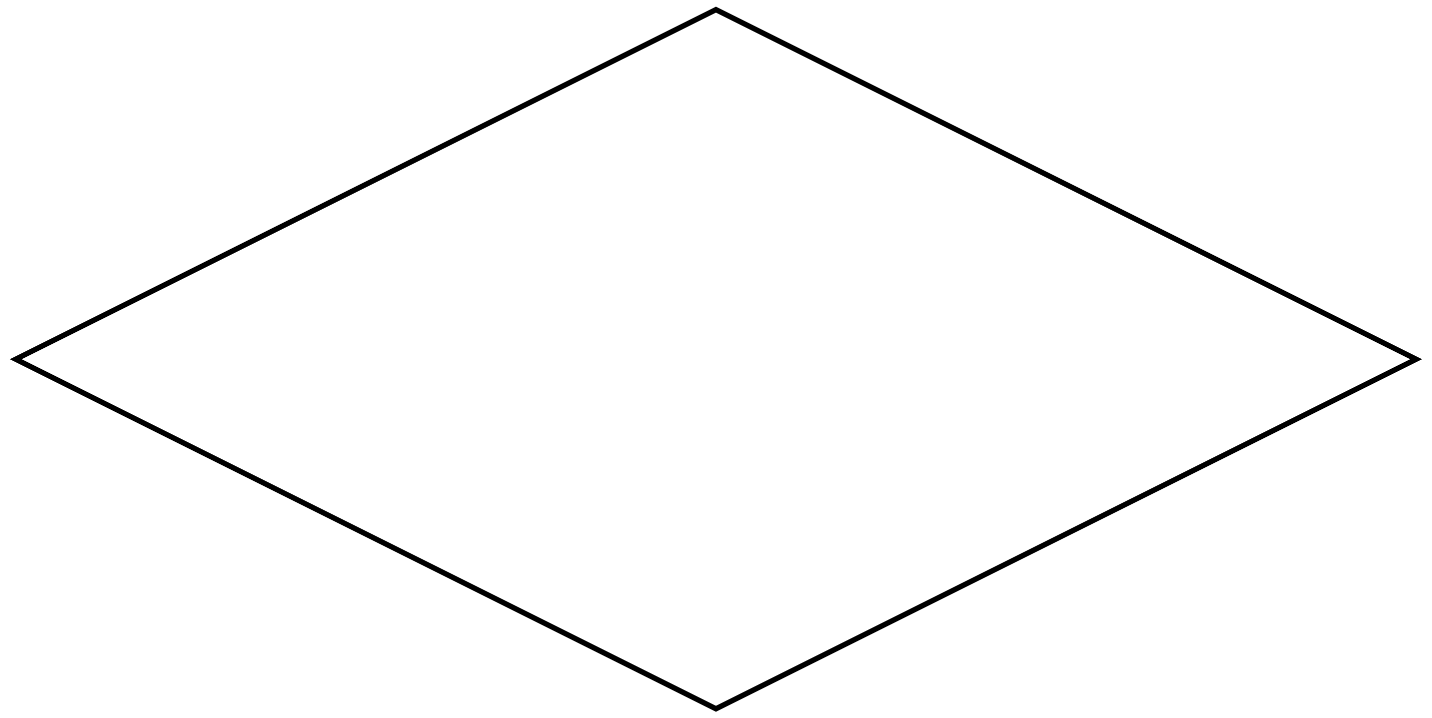
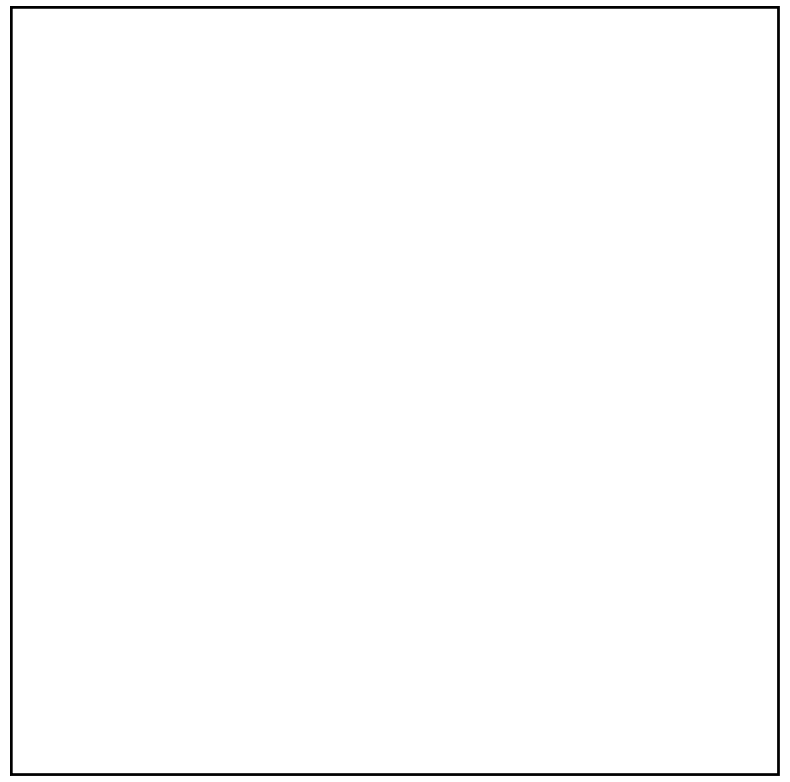
Figure	Definition and Properties	Area of the figure
 **Parallelogram**	Quadrilateral in which opposite sides are parallel. Opposite angles and sides are equal. Diagonals bisect each other.	Base x height
 **Rhombus**	Quadrilateral in which all sides are equal. (Which means opposite sides are equal, and hence it is a parallelogram.) Diagonals are perpendicular to each other. Diagnols are angle bisectors.	1/2 x (product of diagonals)

Figure	Definition and Properties	Area of the figure
 Square	Quadrilateral in which all interior angles and all sides are equal. All properties of rectangle and rhombus It is a regular quadrilateral.	$\text{Side}^2 = 1/2 \times \text{diagonal}^2$

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Polygon

Figure obtained by joining n non-collinear points, where $n > 2$, is called Polygon. n is the number of vertices, and same is the number of sides. A polygon in which all sides are equal and all angles are also equal is called Regular Polygon. Any polygon will satisfy these properties:

1. Sum of all interior angles is $(n-2) \times 180°$.

2. Sum of all exterior angles is $360°$.

3. Number of Diagonals = $n(n-3)/2$.

Common names of polygons: 3 sides-triangle; 4 sides-quadrilateral; 5 sides-pentagon; 6 sides-hexagon; 7 sides-heptagon; 8 sides-octagon; 9 sides-nonagon; 10 sides-decagon.

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Circles

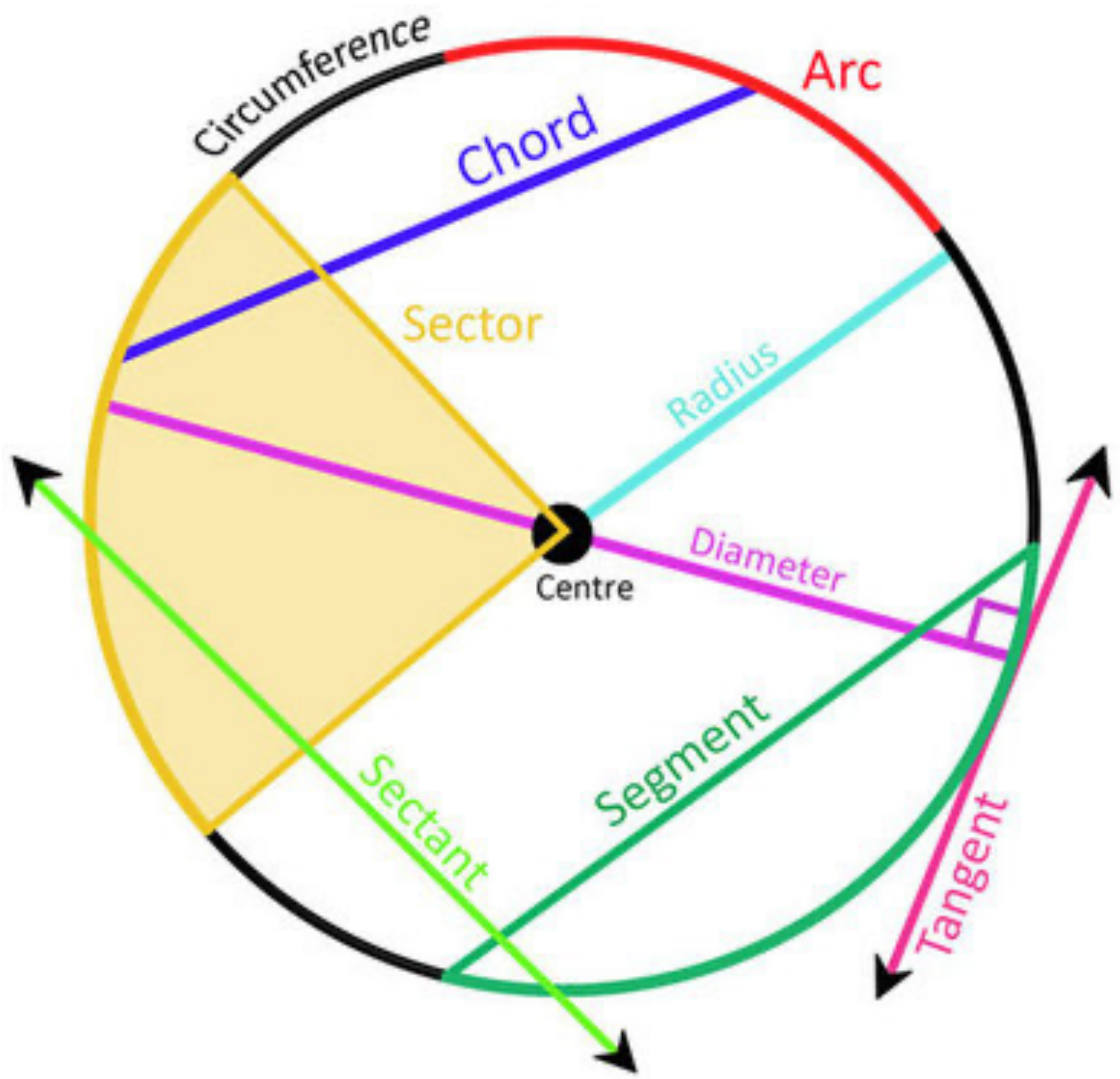

Figure obtained by joining all points which are at a given distance from a fixed point is called circle.

Fixed point is called centre. Distance of the circumference is called radius (OA).

Line segment joining any 2 points on the circle is called chord. Longest chord is diameter. Perpendicular from centre bisects a chord.

Line that intersects the circle at one point is called tangent. Tangent is perpendicular to the radius at point of tangency.

Segment of a circle is called arc. Area formed by an arc and 2 radii at the endpoints of it is called sector. ϴ is the angle subtended by the arc at the centre.

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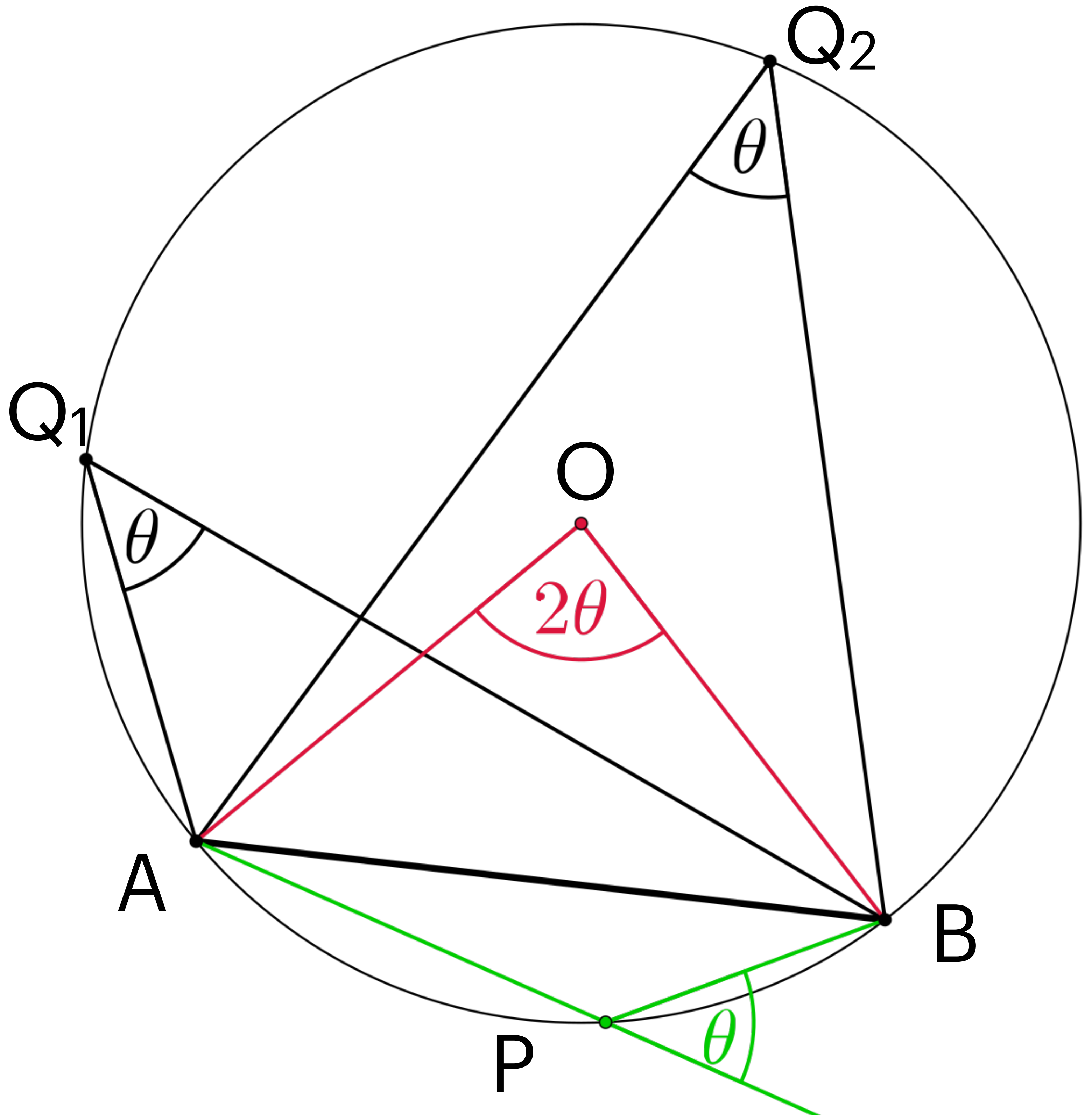
Arc is part of a circle. For e.g. let APB be an arc of the circle. Angle subtended by an arc at the centre from any point on circle is always twice the angle subtended on opposite arc from same two points.



Let arc APB subtends angle 2ϴ at the centre.
Then arc APB subtends ϴ anywhere in arc AQ_1Q_2B.

Angle inside a semicircle is 90°. Property can also be said as: If a circle is drawn passing through all vertices of a right triangle, then hypotenuse is the diameter of that circle. Hence, midpoint of hypotenuse is the centre of the circle.

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Formulae

Circumference of a circle = $2\pi r$

Area of a circle = πr^2

Length of the arc = $\theta/360 \times 2\pi r$

Area of a sector = $\theta/360 \times \pi r^2$

Perimeter of a sector = length of the arc + length of 2 radii at the end points.

3-Dimensional Geometry

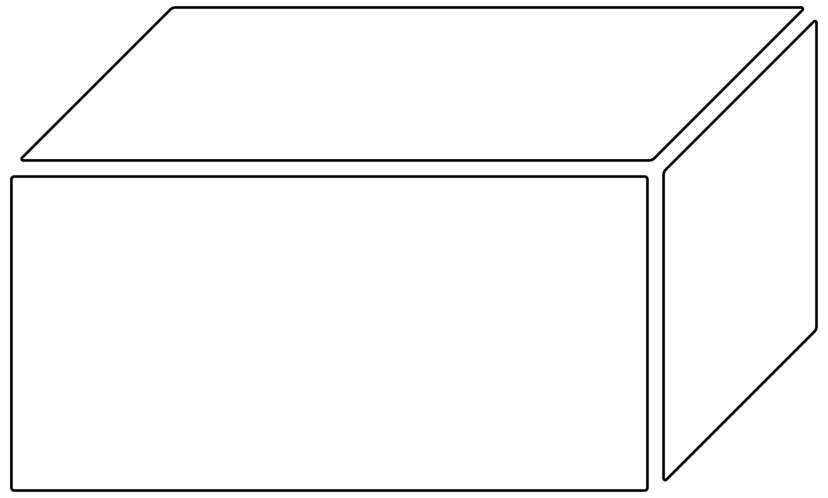
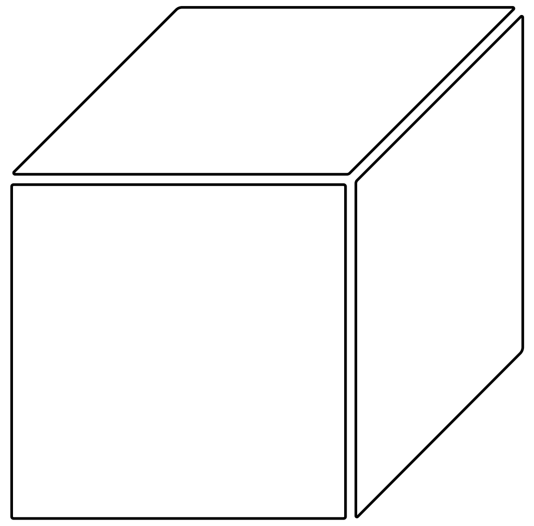
Figure	Terminology	Volume	Surface Area (CSA: Curved Surface Area; TSA: Total Surface
 **Cuboid**	l= length b= breadth h= height d= diagonal d= $\sqrt{l^2 + b^2 + h^2}$ (Body diagonal)	L X b X h	TSA= 2(lb+bh=lh)
 **Cube**	a= edge length d= $\sqrt{3}a$ (Body diagonal)	a^3	TSA= $6a^2$

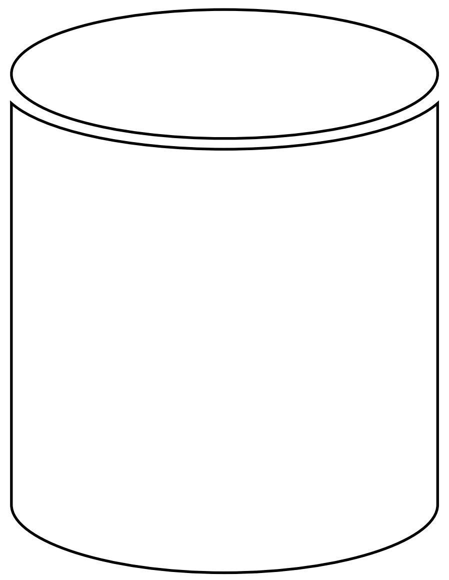
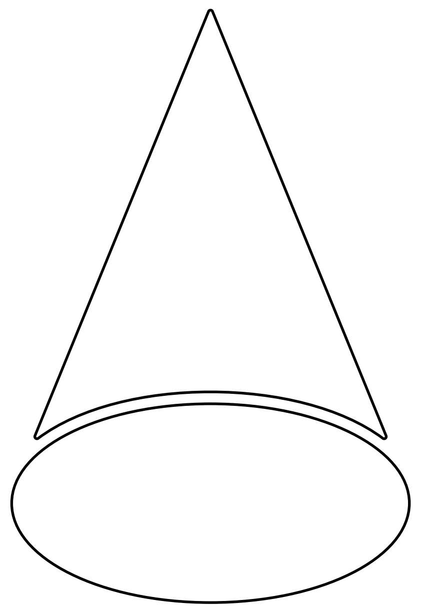
Figure	Terminology	Volume	Surface Area (CSA: Curved Surface Area; TSA: Total Surface
 **Cylinder**	r= base radius h= height	$\Pi r2h$	CSA= $2\pi rh$ TSA= $2\pi r(h+r)$
 **Cone**	r= base radius h= height $l= \sqrt{r^2 + h^2}$ (Slant height)	$1/3r^2h$	CSA= πrl TSA= $\pi r(l+r)$

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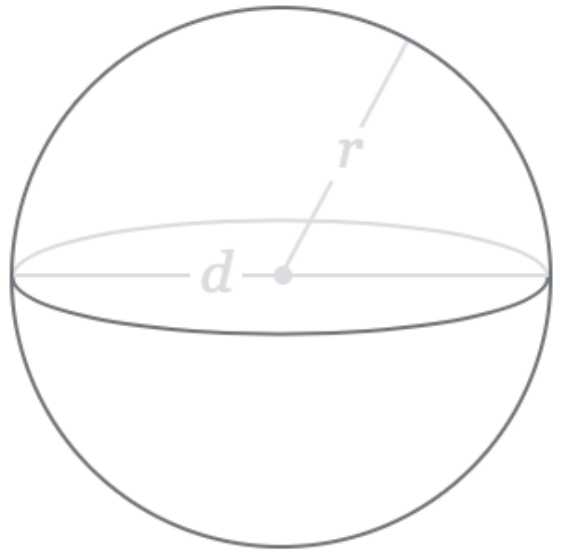
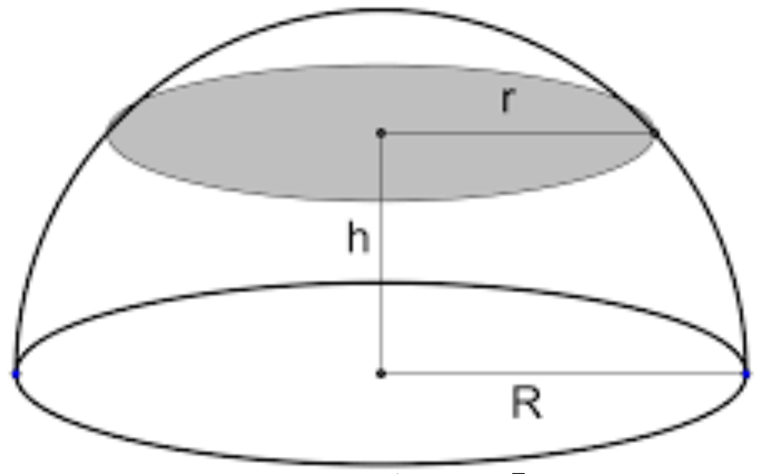
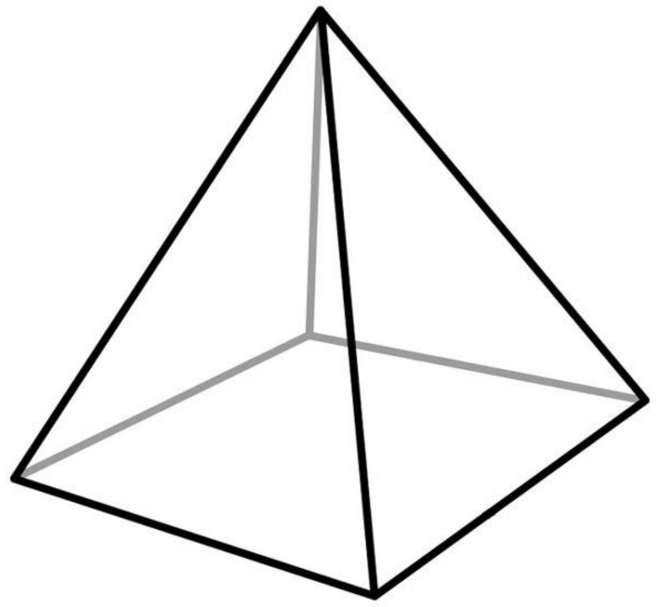
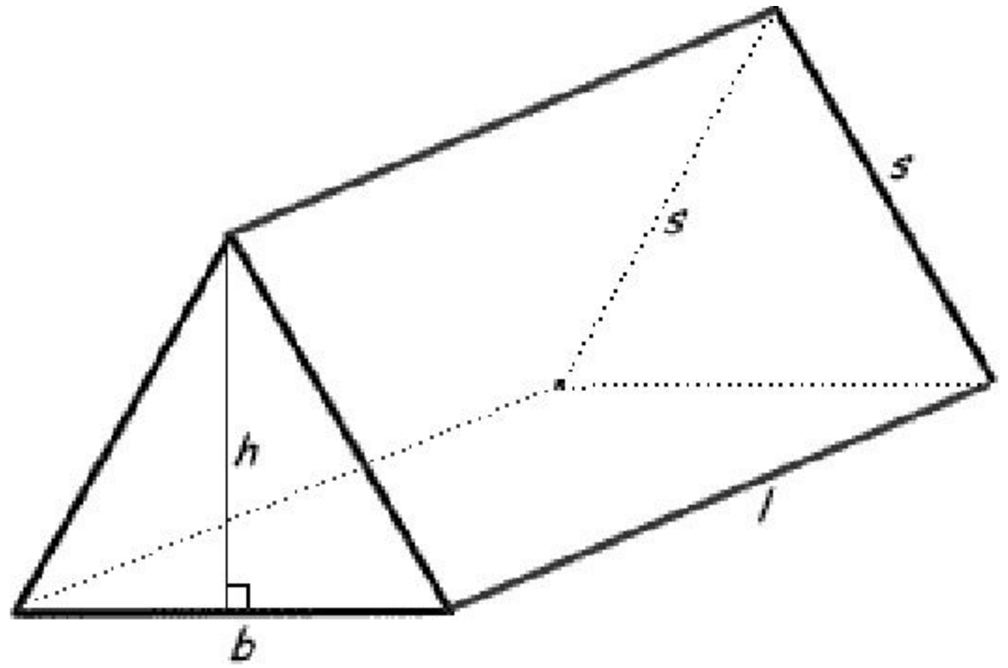
Figure	Terminology	Volume	Surface Area (CSA: Curved Surface Area; TSA: Total Surface
 **Sphere**	r= radius	$4/3\pi r^3$	TSA= $4\pi r^2$
 **Hemisphere**	r= radius	$2/3\pi r^3$	CSA= $2\pi r^2$ TSA= $3\pi r^2$

Figure	Terminology	Volume	Surface Area (CSA: Curved Surface Area; TSA: Total Surface
 **Pyramid**	r= radius	$4/3\pi r^3$	TSA= $4\pi r^2$
 **Prism**	h= height S, b= sides of the triangle l= length	V= 1/2(bhl)	TSA = 6h + 2ls + lb

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Coordinate Geometry

Theory

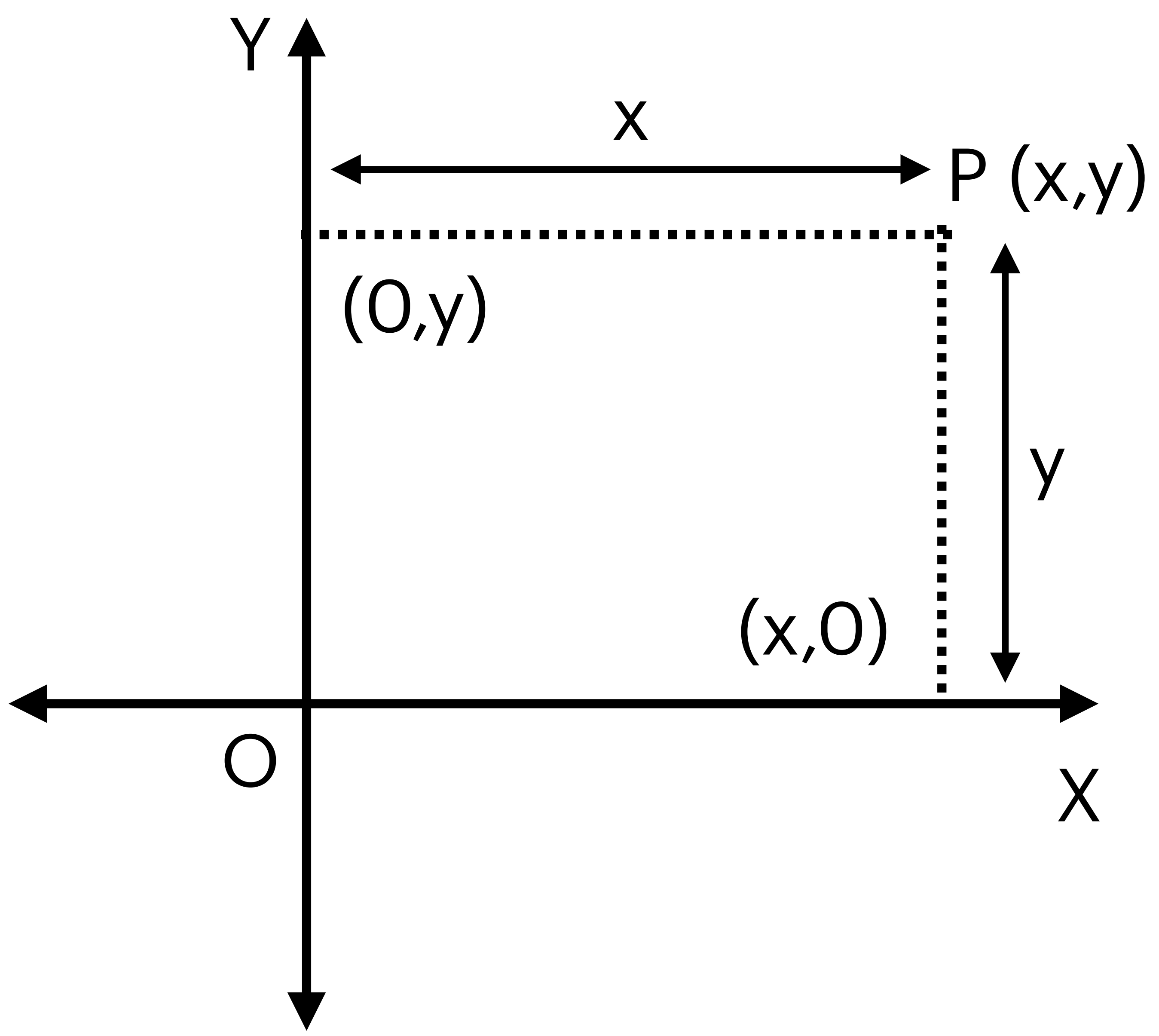
Coordinate geometry is significant in analysing the relation between two parameters and see variation of one quantity with another. Most popular coordinate system is rectangular coordinate system. In a plane, two perpendicular lines are drawn (consider these lines as two number lines) called as x-axis and y-axis (together they are called coordinate axes and the plane is called coordinate plane). Point of intersection of coordinate axes is called Origin (denoted as O).

Location of any point can be specified in terms of two numbers:

First one is x-coordinate, which is distance of the point from y-axis (horizontal distance); and

Second one is y-coordinate, which is distance of the point from x-axis (vertical distance).

x and y coordinates are not just distances actually, they are taken with sign. Any point P can be denoted as (x,y). For eg point (3,4) is at a distance of 3 units from y-axis on the right side, and 4 units from x-axis above it.



For any point in the coordinate plane:

On the right side of origin, x-coordinate is positive;

On the left side of origin, x-coordinate is negative;

On y-axis, x-coordinate is ZERO.

Above the origin, y-coordinate is positive; and

Below the origin, y-coordinate is negative.

On x-axis, y-coordinate is ZERO.

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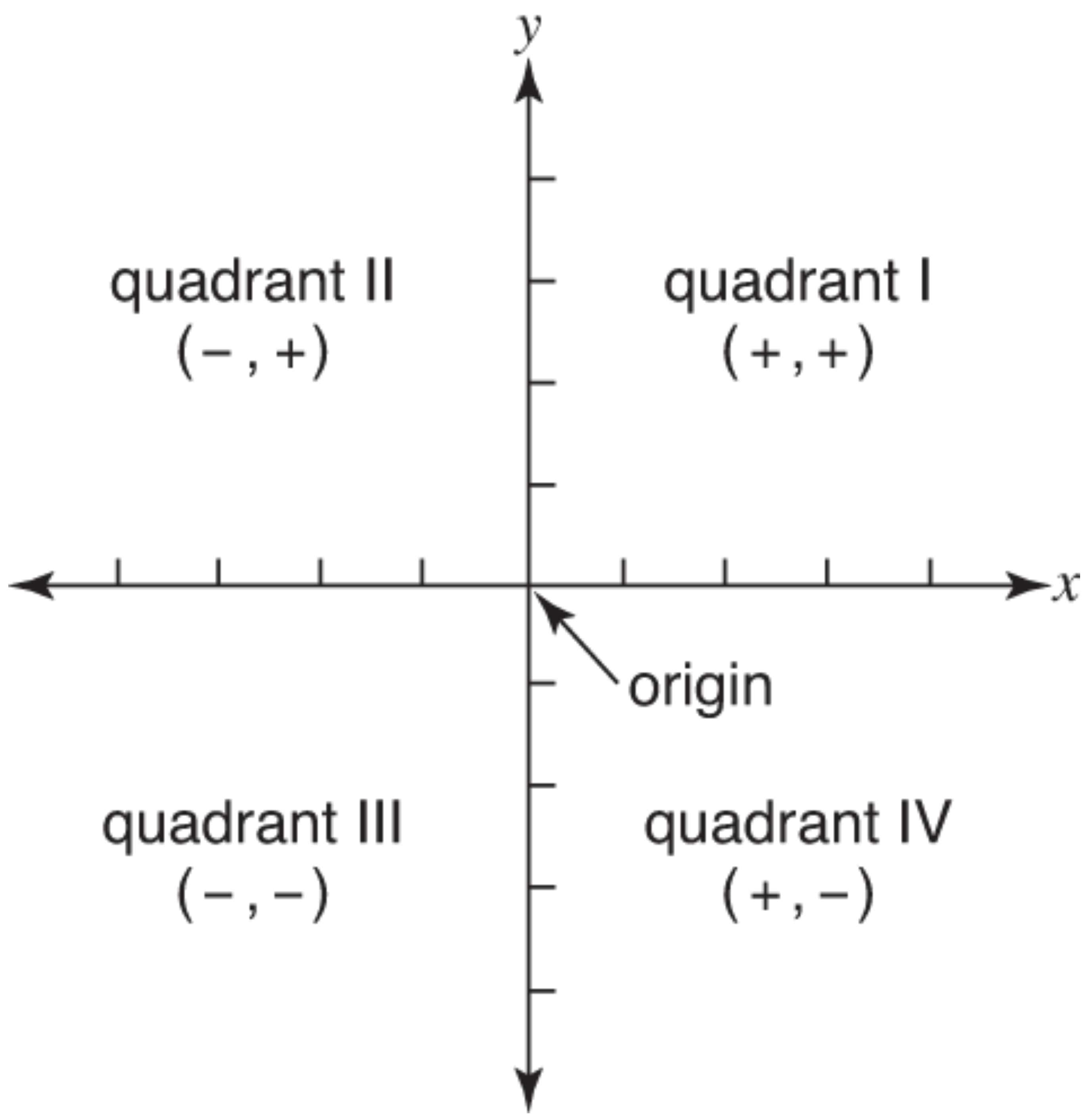
Hence, the coordinate plane can be divided into four regions, called Quadrants, as follows:

1. I Quadrant: Area above x-axis and on right side of y-axis.

2. II Quadrant: Area above x-axis and on left side of y-axis.

3. III Quadrant: Area below x-axis and on left side of y-axis.

4. IV Quadrant: Area below x-axis and on right side of y-axis.



For example, (3,2) lies in 1st Quadrant, (-2,5) lies in 2nd Quadrant, (-1,-4) lies in 3rd Quadrant, and (6,-2) lies in 4th Quadrant.

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Straight lines

Distance formula:

Say points $P(x_1,y_1)$ and $Q(x_2,y_2)$ are given, then:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

i.e. √[difference of x-coordinates]^2 + [difference of y-coordinates]^2

Midpoint formula:

Say points $P(x_1,y_1)$ and $Q(x_2,y_2)$ are given, then:

$$x_1 + x_2 / 2 , y_1 + y_2 / 2$$

Slope:

Slope is inclination of a line with the positive x-axis. It is denoted by m and calculated as:

$$m = (y_2 - y_1)/(x_2 - x_1) = \text{Slope} = \tan(\text{theta})$$

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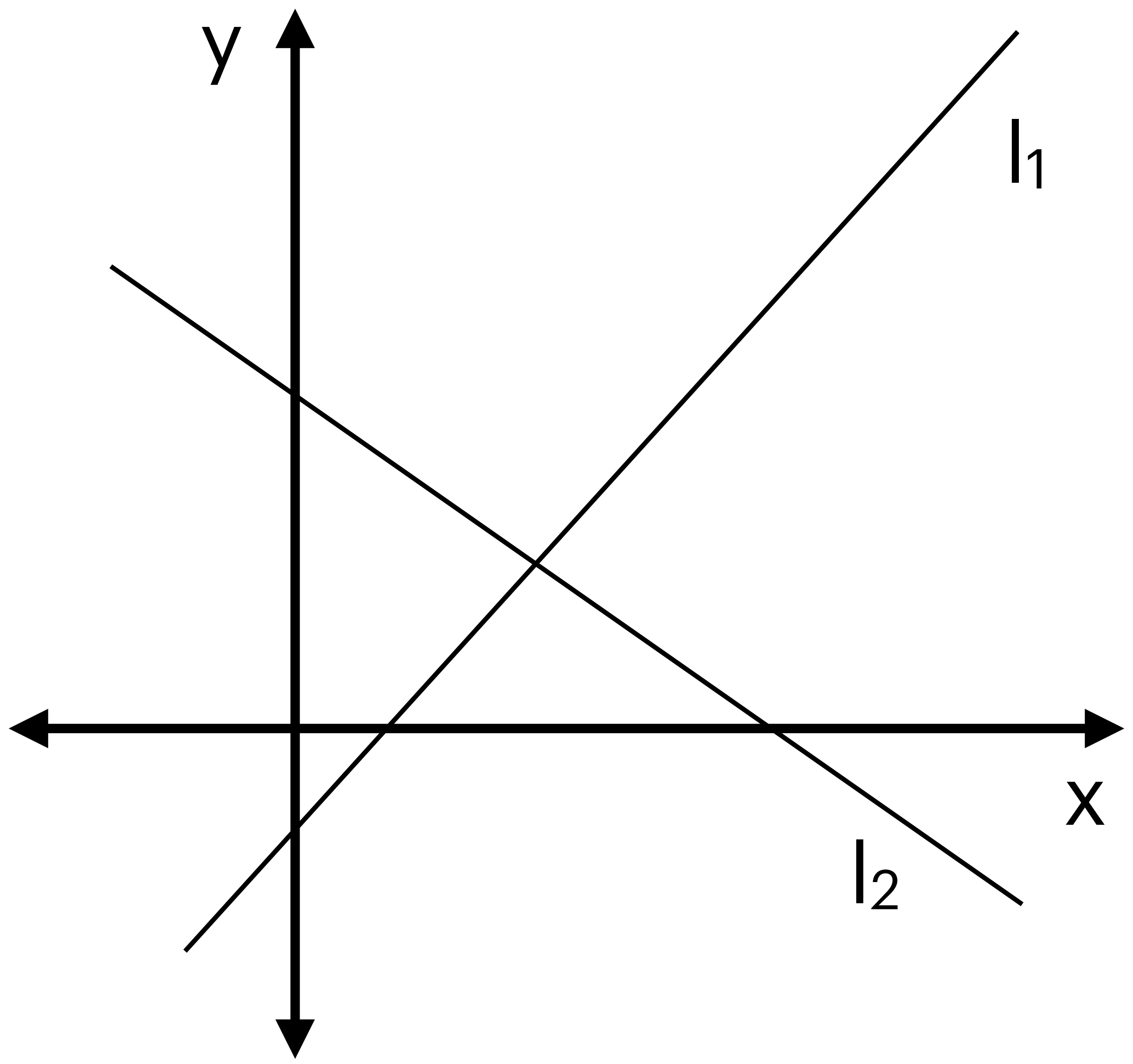
If line is ascending (line l_1), then slope is positive, i.e., $m > 0$.

If line is descending (line l), then slope is negative, i.e., $m < 0$.

If line is || to x-axis, then slope is ZERO, i.e., $m = 0$.

If line is || to y-axis, then slope is NOT DEFINED.

When the lines are parallel, their slopes are equal, i.e., $m_1 = m_2$.



When the lines are perpendicular, then the product of their slope is -1, i.e., $m_1 \times m_2 = -1$.

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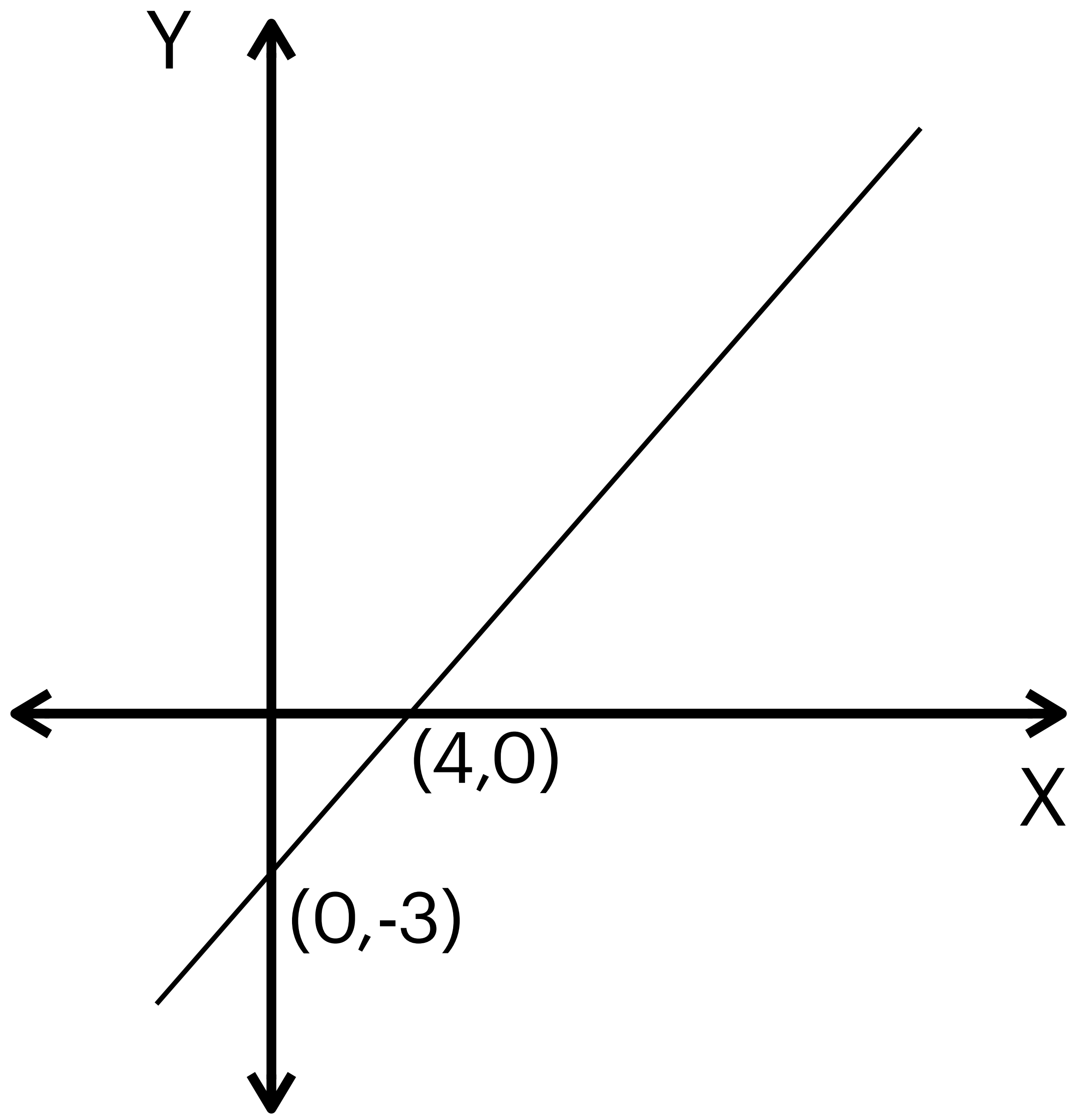
Intercepts:

X-intercept is x-coordinate of the point where the line intersects x-axis. Similarly, y-intercept is y-coordinate of the point where the line intersects y-axis.
For example, in the following figure, x-intercept is 4 and y-intercept is -3.



If a line passes through origin, then both x-intercepts and y-intercepts are ZERO.

A line has x-intercept ‘a’ and y-intercept ‘b’. At x-intercept y coordinate of the point will be 0, and, at y-intercept, x coordinate of the point will be 0.
Hence, the points can be represented as (a,0) and, (0,b), respectively.

Equation of a line:

Equation of the line is a relation between x-coordinate and y-coordinate for any point on the line. Hence, all points on the line will satisfy this relation. In general, equation of a line is:

Formulae:

1. $y=mx+c$
2. $y_2-y_1 = m(x_2-x_1)$
3. $x/c + y/b = 1$

Where m is the slope, and c is the y-intercept of the line.

Therefore, to find the equation of any line, m and c have to be calculated. Slope (m) and y-intercept (c) of a line are constant, i.e., one line will have only one value of slope and one value of y-intercept.

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For example, Find the equation of the line that has slope 2 and y-intercept 4. **Solution**: Slope, i.e., m=2 and y-intercept, i.e., c=4.
Hence, equation of the line is: $y = 2x + 4$, i.e., all points on this line will satisfy this relation. So, y-coordinate is equal to twice of x-coordinate plus 4 for every point on the line. So by using this relation, if x-coordinate of some point is given (say 5), then y-coordinate of that point can be circulated ($2 \times 5 + 4$, i.e., 14) or vice-versa. Hence, point (5,14) lies on this line.

NOTE:

Any line || to x-axis is of the form: **y = c, i.e., y = (some constant).**
For example, y=4 is a line || to x-axis passing through points (0,4), (1,4),..etc. Hence, y-coordinate will always remain 4 on this line.

Any line || to y-axis is of the form: **x = k, i.e., x = (some constant).**
For example, x=3 is a line || to y-axis passing through points (3,0), (3,1),...etc. Hence, x-coordinate will always remain 3 on this line.

Equation of a circle:

Circle is the set of all points in a plain that are at a given distance from a point, that is the centre. The distance between any of the points and the centre is called the radius.

The equation of the circle with centre (a,b) and radius r is:

$$(x-a)^2 + (y-b)^2 = r^2$$

Also, the equation of the circle with centre as origin and radius r is
$x^2 + y^2 = r^2$

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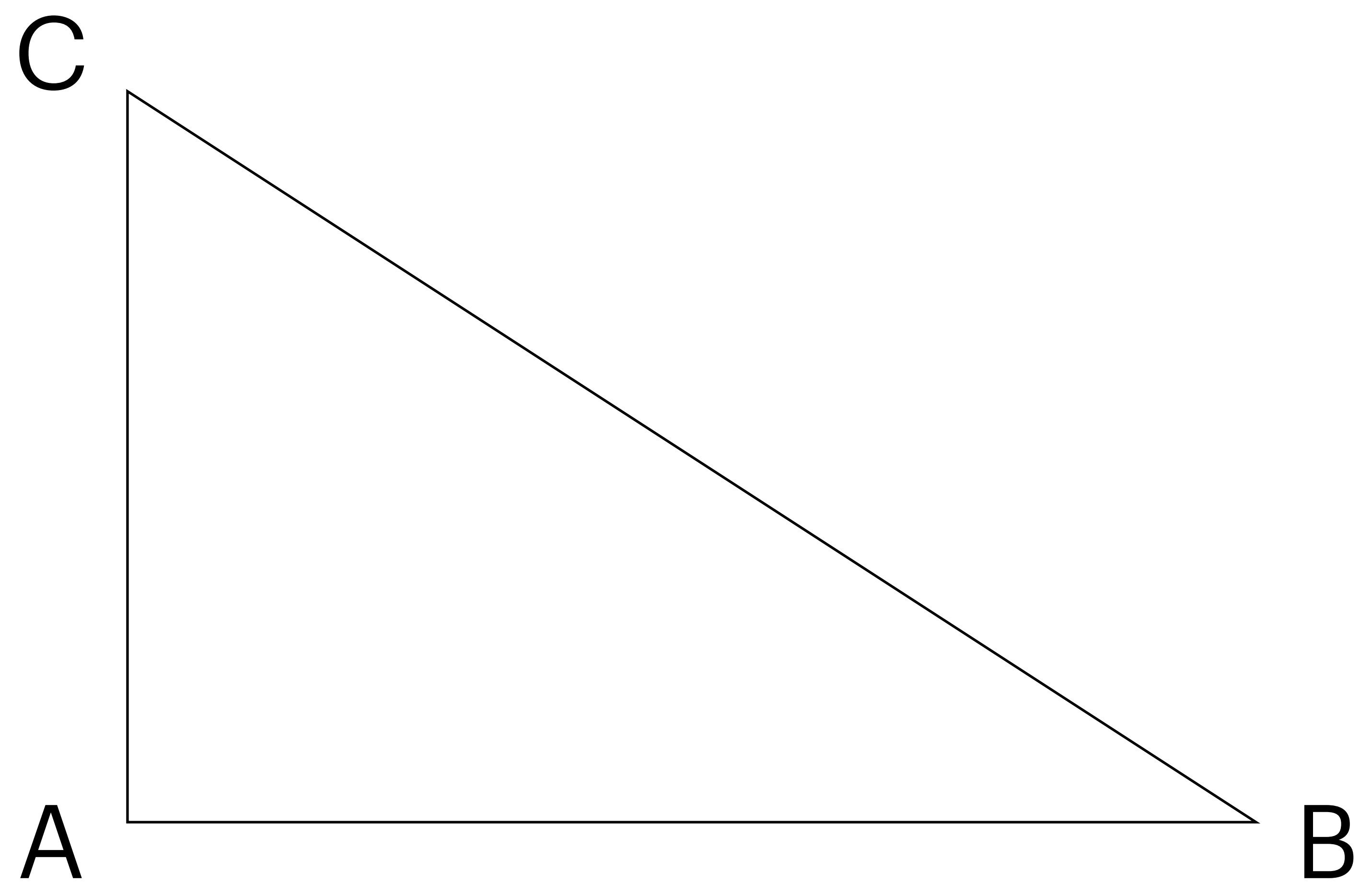
Trigonometry + Radians

Theory

In triangle ABC, Angle A is 90°. We can define Sine, Cosine, Tangent (these functions are abbreviated as sin, cos, tan respectively) using this triangle.



For angle B, Hypotenuse is the largest side, Base is the side adjacent to angle B, and Perpendicular is the side opposite to angle B.	For angle B, Hypotenuse is the largest side, Base is the side adjacent to angle C, and Perpendicular is the side opposite to angle C.
Sin (B) = Perpendicular/Hypotenuse = AC/BC	Sin (B) = Perpendicular/Hypotenuse = AB/BC
Cos (B) = Base/Hypotenuse = AB/BC	Cos (B) = Base/Hypotenuse = AC/BC
Tan (B) = Perpendicular/Base = AC/AB	Tan (B) = Perpendicular/Base = AB/AC

Angles measured in degrees and in radians, both.

We have, $360° = 2\pi$ radians,

$1° = 2\pi/360$ radians

$30° = \pi/6$ radians

$45° = \pi/4$ radians

$60° = \pi/3$ radians

$90° = \pi/2$ radians

$180° = \pi$ radians

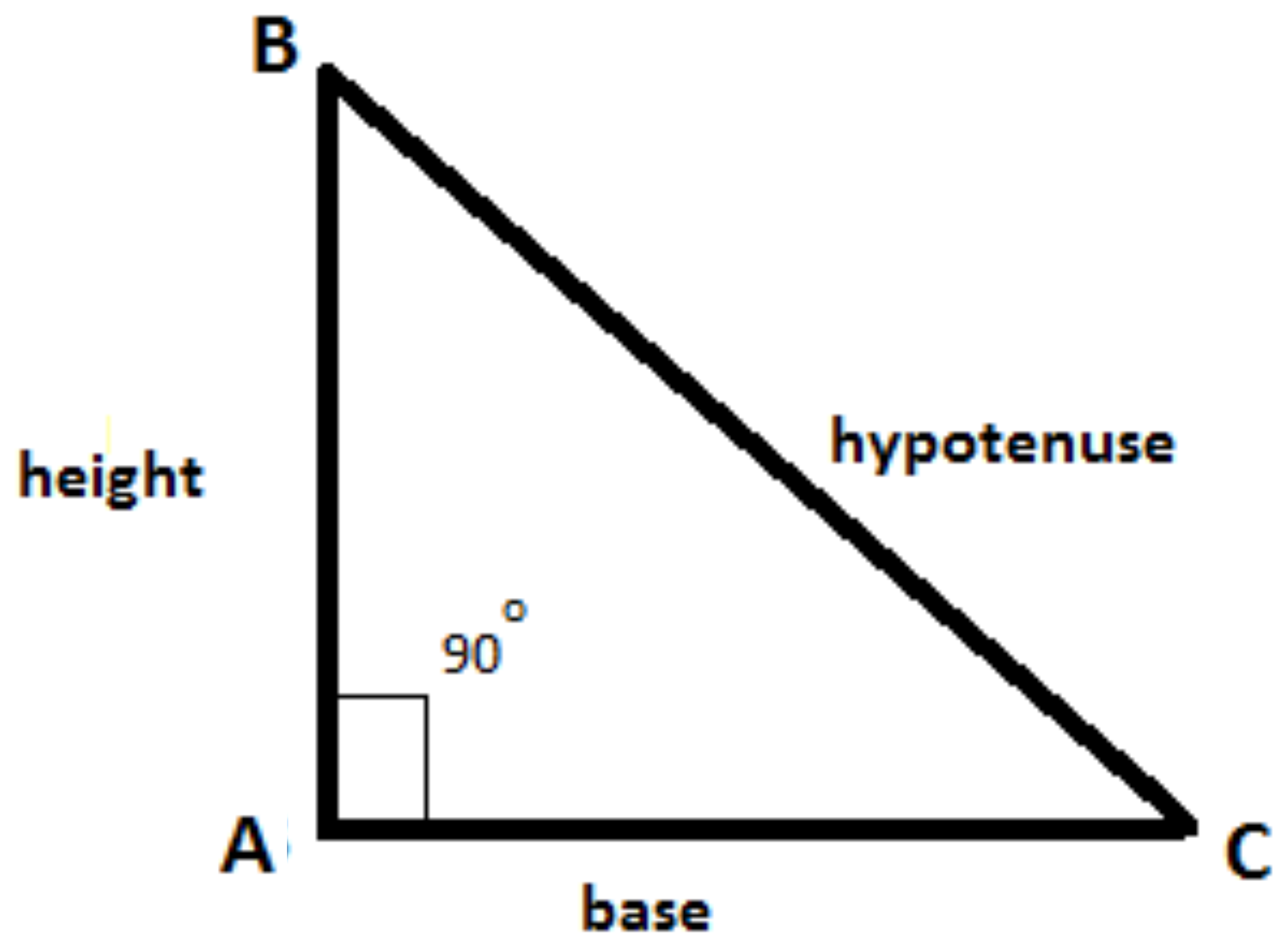
$270° = 3\pi/2$ radians

(Measure in radians)/(measure in degrees) = $2\pi/360°$ radians

In triangle ABC,

Here, Angle A is 90°, the length of AB is 4 units, and the length of AC is 3 units.

So, using Pythagoras theorem we can find that the length of BC (hypotenuse) will be 5 units.



Sin B = 3/5	Sin C = 4/5
Cos B = 4/5	Cos C = 3/5
Tan B = 3/4	Tan C = 4/3

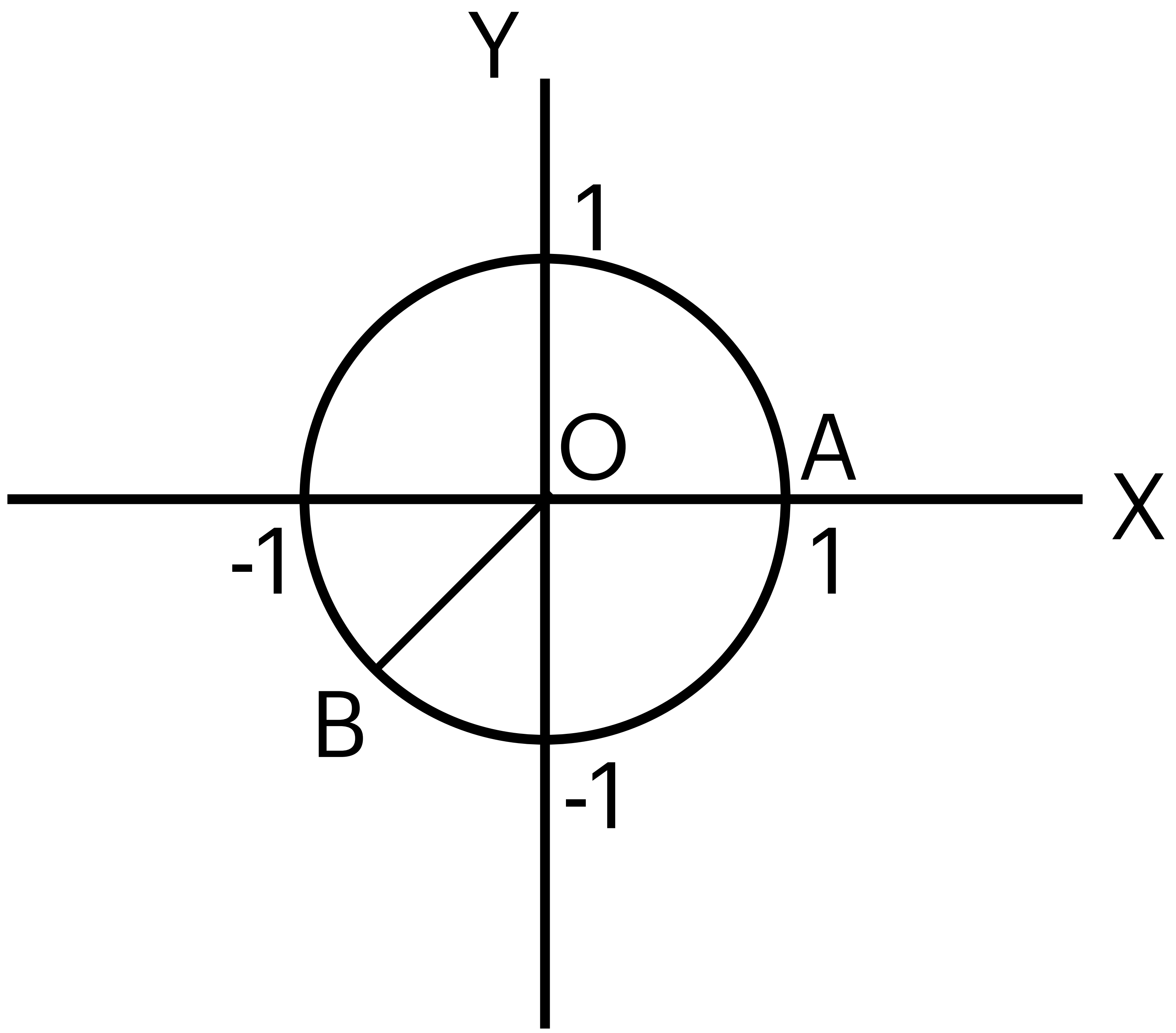
As we can observe,

Sin B = cos C = 3/5	Sin C = cos B = 4/5
Sin B = cos (90°- B)	Sin C = cos (90°- C)
Sin B = cos (π/2 - B)	Sin C = cos (π/2 - C)

Also, tan (x) = Perpendicular/Base
= (Perpendicular/Hypotenuse)/(Base/Hypotenuse)

Or, tan (x) = sin x/cos x

For all angles between 0 and π/2, we can use a right angled triangle to get trigonometric ratios, for other angles we will use a circle of radius 1 with centre as origin. The circumference of the circle is equal to 2πR, which is equal to 2π.



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Let us consider the trigonometric ratios in the circle of radius 1 and centre O as origin.
Here, radius is equal to hypotenuse (OB) 1.

Sin of angle BOA is the value of y coordinate of point B.

Similarly, cos of angle BOA will be equal to the value of x coordinate of point B.

A complete circle has the angle from 0° to 360° or, from 0 to 2π.

Also, a rotation of 2π about the origin brings us back to the same point, so we have

Sin (x + 2π) = sin (x)

Cos (x + 2π) = cos (x)

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The complete angle inside the circle is 360° = 2π, or angle in radians is equal to the arc length for a circle with unit radius.

If the radius of the circle is R units, then

Hypotenuse = R, $\sin\theta = y/R$, $\cos\theta = x/R$

Also, $l = r\theta$

Where, l= length of the arc, r = radius, θ = Angle (in radians)

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Percentage

Theory

Percent (per cent i.e. per 100), as the word implies, means out of 100, e.g. Adam got 70 marks out of 80, which means he got 70/80 x 100 = 87.5% (70 out of 80 is equivalent to 87.5 out of 100.)

a% of b is c => a/100 x b = c i.e. % means 1/100; of means multiplication; is means equal to.

To convert fraction to %, multiply by 100: e.g. 1/2 = 1/2 x 100% = 50%.

To convert % to fraction, divide by 100: e.g. 25% = 25/100 = 1/4.

Formula

Percentage change = final - initial / initial x 100%

Note: if percentage change is negative, that means quantity decreases.

If a quantity changes by p%, then:

Final value= initial + p/100 x initial = (1 + p/100) x initial.

Note: In case of decrease, p is negative, e.g. 700 decreases by 20%.

=> Final value = 700 - 20/100 x 700 = 700 - 140 = 560

Ratio and Proportion

Theory

Ratio and proportion is the basis of percentages. To form equations and understand variations, one should be thorough with the concepts of ratio and proportion.

Ratio

Ratio is comparison of two or more quantities (units of both quantities have to be same.) For e.g. in a class of 30 students with 10 boys and 20 girls. So, boys and girls are in the ratio of 10:20 or 1:2.

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If a and b are in the ratio 3:5, then assume a as 3k and b as 5k. This method is very useful to solve ratio questions. Similarly, if a:b:c = 3:5:8, assume a = 3k, b = 5k, and c = 8k.

Example:

a and b are in the ratio 4:7. Sum of a and b is 132. Find a.

Solution:

Let a = 4k and b = 7k

a+b = 4k + 7k = 132,

Which gives: k = 12

a= 4k = 4 x 12 = 48

Proportion

Proportion is equality of two ratios, i.e., a:b::c:d => a/b = c/d

Direct Proportion:

A is directly proportional to b,

=> a increases as b increases and vice-versa.

=> a $\propto$ b

=> a = kb (proportionality sign is replaced with equals to and multiplied by a constant.)

=> a/b = k (where k is a constant)

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Inverse Proportion:

a is inversely proportional to b

=> a decreases as b increases.

=> $a \propto 1/b$

=> $a = k/b$

=> $a \times b = k$ (where k is a constant)

Remember: If directly proportional, then RATIO IS CONSTANT; and if inversely proportional, then PRODUCT IS CONSTANT.

Simple Interest and Compound Interest

Theory

Simple Interest:

Say a person invests \$P at the annual rate of R% simple interest. This means every year, R% of \$P is given as interest.

In 1st year: Interest given = P x R/100
In 2nd year: Interest given = P x R/100
In 3rd year: Interest given = P x R/100 ..... and so on.

Hence, total simple interest in N years is addition of all these individual interests.

Formula:

Simple interest: SI = P x R/100 x N

P = Principal amount; R = Rate of interest; N = Number of terms.

Note:

R and N should always have same time unit, i.e.,

If interest is given monthly, N is number of months and R is monthly rate of interest;

If interest is given quarterly, N is number of quarters and R is quarterly rate of interest; and so on.

Total amount is equal to simple interest added to the principal amount.

Formula:

$A = P + SI$

(Total Amount = Principal Amount + Simple Interest)

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Compound Interest:

Say a person invests $P at an annual rate of R% compounded annually. This means that every year, his amount increases by R%

After 1st year: Total amount: $A_1 = P \times (1 + R/100)$

After 2nd year: Total amount: $A_2 = P \times (1 + R/100)^2$

After 3rd year: Total amount: $A_3 = P \times (1 + R/100)^3$

Hence, total amount after N years:

Total amount: $A = P \times (1 + R/100)^N$

Formula:

Total Amount: $A = P(1 + R/100)^N$

P = Principal amount; R = Rate of interest; N = Number of terms.

Note:

If interest is given monthly, N is number of months and R is monthly rate of interest;

If interest is given quarterly, N is number of quarters and R is quarterly rate of interest; and so on.

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Above is the total amount in N periods. Compound interest is calculated as:

Formula:

CI = A + P

(Compound Interest= Total Amount + Principal Amount)

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Inequalities + Absolute Values

Theory

Basic idea in solving inequalities is same as that of equations, i.e., isolate variable on one side and then simplify. We suggest one should understand the meaning of the inequality first and then solve.

Example

Find the interval in which x lies: $7x + 20 > 40 - 3x$

Solution

$7x + 3x > 40 - 20 \Rightarrow 10x > 2 \Rightarrow x > 2$

Note:

If a positive number is multiplied to both sides of inequality, then inequality remains same; but if a negative number is multiplied to both sides of inequality, then inequality reverses.

For e.g.: $2 < 3$

- If 10 is multiplied to both sides, then inequality remains same, i.e. $20 < 30$
- If - 10 is multiplied to both sides, then inequality reverses, i.e. $-20 > -30$

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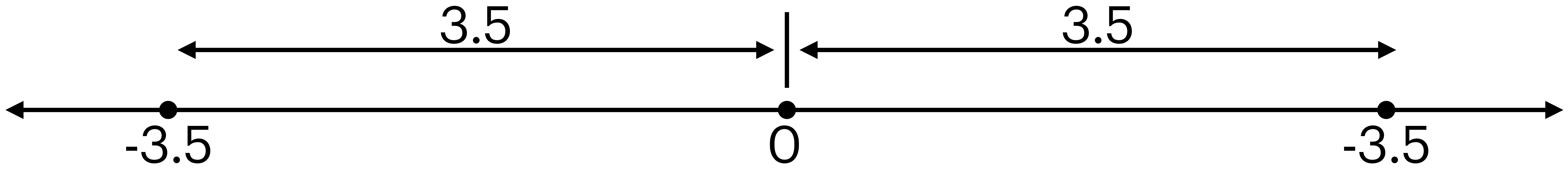
Modulus/Absolute Value

Modulus of any number denotes the distance of that number from zero on number line. For e.g.: | 3.5 | is 3.5 because the number 3.5 lies at a distance of 3.5 units from zero. | - 3.5 | is also 3.5 because the number -3.5 is also at a distance of 3.5 from zero.



Linear Equations

Theory

An important aspect in solving any mathematical problem is to form equations and solve them.

<u>Equations with one unknown variable</u>

Step 1: Assign a variable to the unknown quantity.
Step 2: Form the mathematical equation.
Step 3: To solve the equation, isolate the variable on one side and remaining values on the other side.

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Equations with two unknown variables

Step 1: Assign variables to the unknown quantities
Step 2: Form the mathematical equations.
Step 3: To solve the equations, isolate variables on one side and remaining quantities on the other side. Then eliminate one of the variables.

Example: As store sells pens for $5 each and pencils for $4 each. If John buys a total of 10 pens and pencils for $44, how many pens did he buy?

Solution:
Step 1: In above problem, number of pens and pencils bought is unknown. Let these quantities be x and y respectively.

Step 2: Total number of pens and pencils John bought: $x + y = 10$ .... (1)

Since cost of a pen is \$5 and cost of a pencil is \$4, he has spent an amount of $5x$ on pens and $4y$ on pencils. Total amount spent by him is \$44. Hence:

$5x + 4y = 44$ .... (2)

Step 3: Eliminate one of the variables (say y). Multiplying equation (1) by 4:

$4x + 4y = 40$ .... (3)

To eliminate y, subtract equation (3) from (2):

$(5x + 4y) - (4x + 4y) = 44 - 40$

=> $5x - 4x = 4$
=> $x = 4$, i.e., Humber of pens John bought is 4.

A smarter way

Remember: Too many variables take too much time to solve. Hence, try to solve the question by assuming only one unknown. Often a systematic approach and the use of a tabular method to frame the equations helps to solve problem efficiently.

Since the number of pens John bought is to be calculated, so let it be x

	Pens		Pencils	
Quantity	x		10 - x	
Cost of each	$5		$4	
Total cost	5x	+	4(10 - x)	= 44

=> 5x + 40 - 4x = 44

=> 5x - 4x = 44 - 40

=> x = 4, i.e., Number of pens John bought is 4

Special Equations

Sometimes a question might involve two unknowns, but only one equation is given. In such a case, if may ask for one possible value, not a definite answer. Or it may ask for a ratio, which can be calculated by simplifying the given equation. Take a look at the following example.

Example: $a + b/a = 10$. Find a/b.

Solution: Cross multiplication gives: $a + b = 10a$

=> $b = 10a - a$
=> $b = 9a$
=> $a/b = 1/9$

System of equations:

The relationship among Linear Equations:

A linear Equation in two variables can be solved by plotting them on the coordinate plane. The point of intersection of the two linear equations is the solution to the system of equations.

If two linear equations are plotted on the graph they will represent two straight lines. The nature of these lines describe the solution to these equations. The key points are as follows:

1. If the two lines intersect at a point, then the system has a unique solution(Case 1)
2. If the two lines are parallel, then the system has no solution. (Case 2)
3. If the two lines are coinciding, i.e., identical lines, then the system has infinitely many solutions. All the points on these lines will satisfy the equations. (Case 3)

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For solving the system of equations, we can convert the equations into slope-intercept form. This will be helpful in solving Case 2 and Case 3.

If the lines have equal slopes and different y-intercepts, the lines will be parallel.

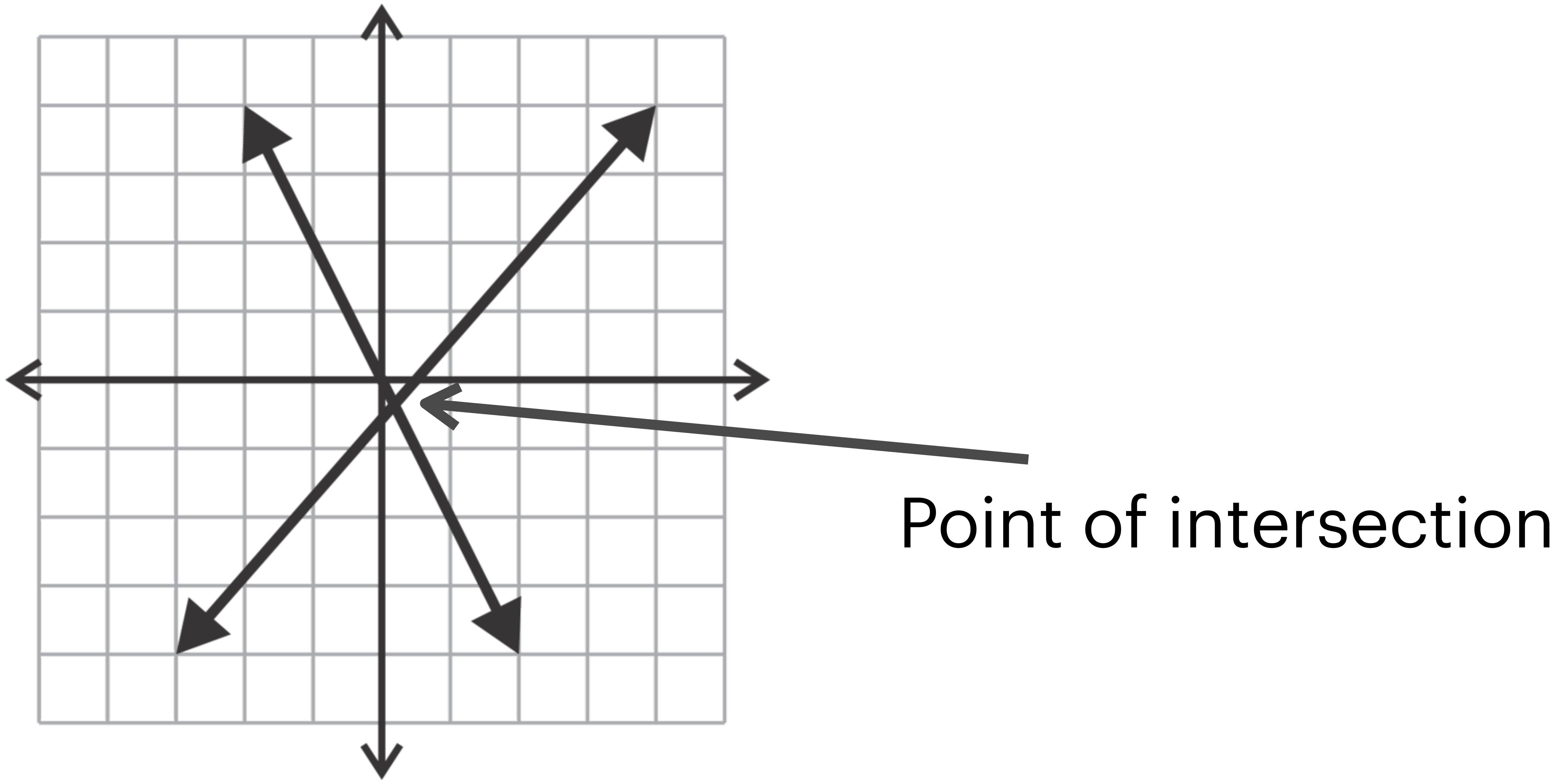
If the lines have equal slopes and same y-intercepts, the lines will be identical.

For a system of equations: $a_1x + b_1y = c$; $a_2x + b_2y = c_2$

Unique solution:

If the system of equations has a unique solution, then the graph will be similar to

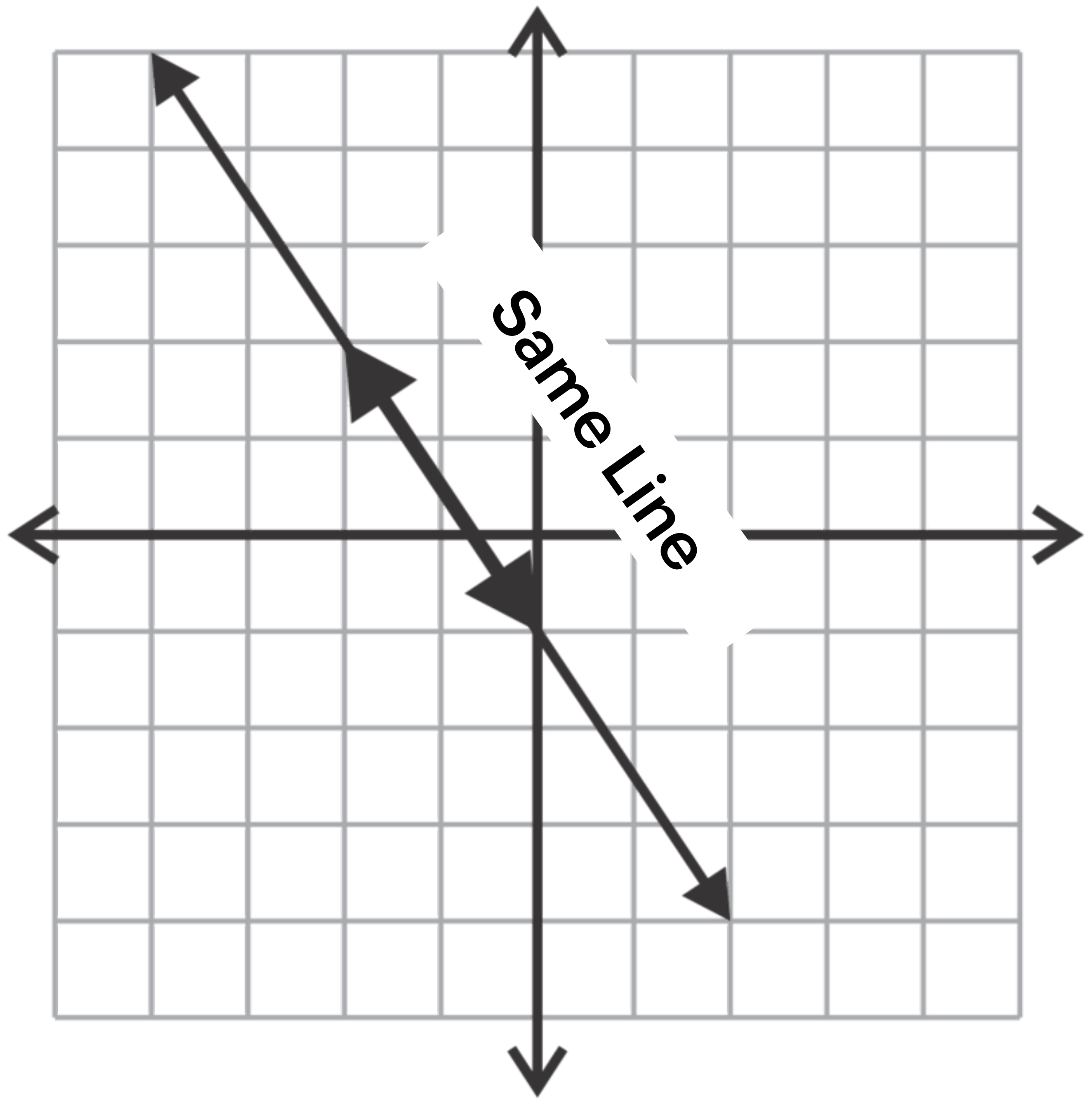
Formula: $a_1/a_2 = b_1/b_2$



Infinitely Many Solutions:

If the system of equations has infinitely many solutions, then the graph will be similar to

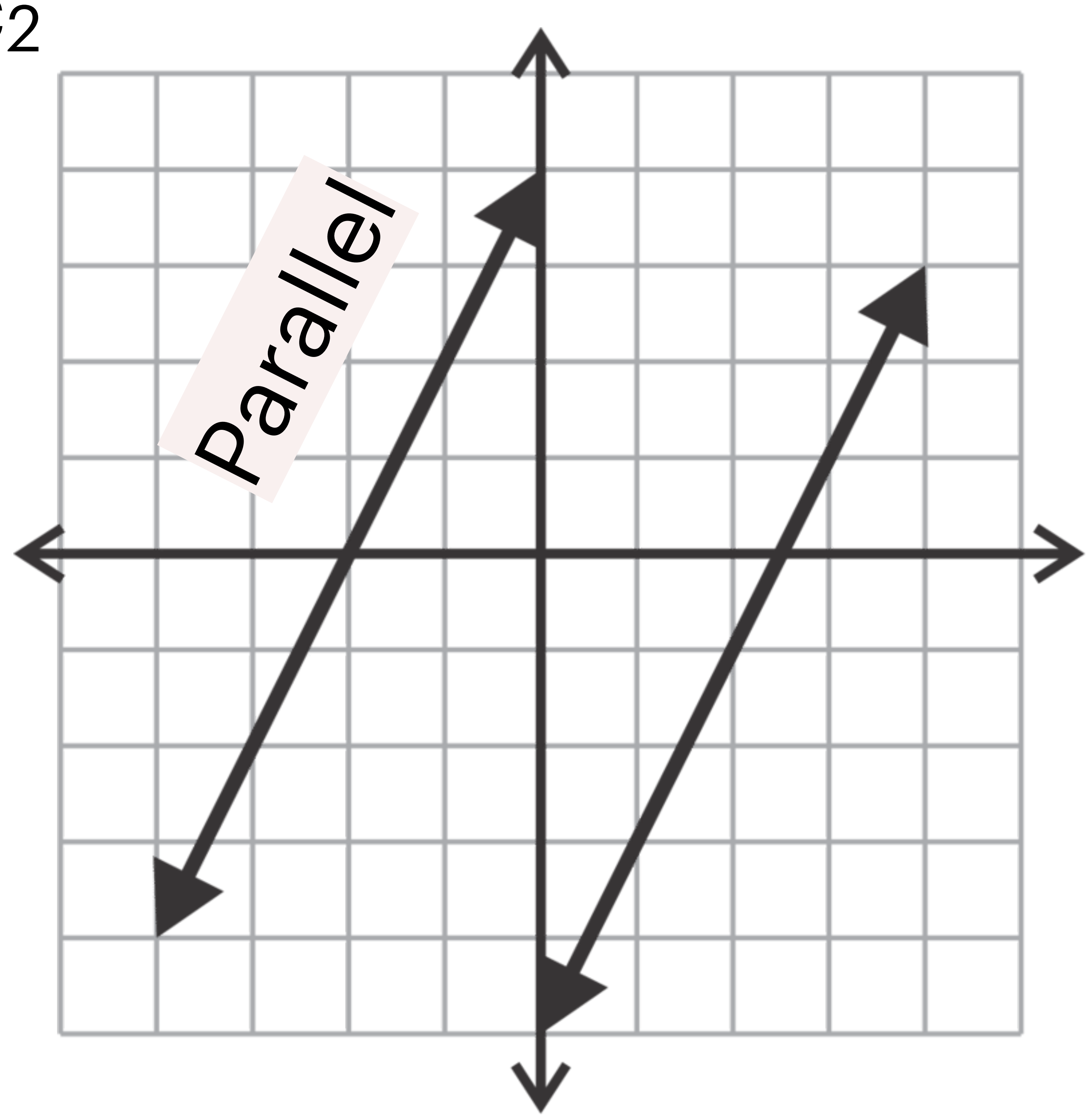
Formula: $a_1/a_2 = b_1/b_2 = c_1/c_2$



No solution:

If the system of equations has no solution, then the graph will be similar to

Formula: $a_1/a_2 = b_1/b_2 \neq c_1/c_2$



Functions and Polynomials

Theory

Quadratic Equations

Quadratic equations can be solved in two ways:

Way 1: Using factorisation

Write the equation as $ax^2 + bx + c$. Then split the middle term into two terms, whose sum is equal to the product of first term and last term.

Example- Solve: $x^2 + 5x + 6 = 0$

=> $x^2 + 2x + 3x + 6 = 0$ (here, we have split 5x into 2x and 3x as 2x+3x = 5x and the product of 2x and 3x is equal to $6x^2$)

=> $x(x + 2) + 3(x + 2) = 0$ (take whatever is common in the 1st two terms and last two terms.)

=> $(x + 2) + (x + 3) = 0$

=> $x + 2 = 0$ or $x + 3 = 0$

=> $x = -2$ or $x = -3$

Way 2: Using Formula

If we are not able to split the middle term then we can use the other way to solve a quadratic equation i.e., using the formula.

Roots (or solutions) of a quadratic equation of the form $ax^2 + bx + c$ are:

Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note: if $b^2 - 4ac$ is negative, then roots don't exist, i.e., given equation has no real solution.

Equations with Square Root

To solve equations with square root, take the quantity with square root on one side and then square both sides.

Example

Solve for t: $\sqrt{t-2} = 4$

Solution Squaring both sides gives: t - 2 = 16 => t = 18

A quadratic equation when plotted on a graph forms a parabola. The following is true for any function f(x) and any positive integer k:

- The graph of f(x) + k is the graph of f(x) shifted upward by k units.
- The graph of f(x) - k is the graph of f(x) shifted downward by k units.
- The graph of f(x + k) is the graph of f(x) shifted to the left by k units.
- The graph of f(x - k) is the graph of f(x) shifted to the right by k units.

Functions

Functions is something that will take an input (usually denoted by x) and based on that input it will produce exactly one output (usually denoted by f(x) or y).

INPUT -> FUNCTION -> OUTPUT

A function can be denoted by an expression like f(x). x is the most commonly used symbol for an input, although other variables like y, z, etc. can also be used. 'f' is the most commonly used symbol for the name of the function. Although other variables like g and h are also used.

For e.g. let a function be defined as $f(x) = 3x + 1$, then what happens if we input 2 into the function?

The way we denote by putting 2 is that we want to evaluate f(2), i.e., replace x by 2.

$f(2)$ is equal to $3(2) + 1 = 7$

This means that when we input 2 in the given function, the output we get is 7.

Composite Functions

A composite function is a combination of two functions, such that the output of one of the functions becomes the input for the other functions.

A composite of functions would be written as:

f(g(x)), (f of g of x), or (fog)(x) (f follows g(x)).

This composite function means that for a given input, or for given x, the output of the function g(x) serves as the input to the function f(x).

For e.g. two functions f(x) and g(x) are defined such that f(x) = 8x and g(x) = 2x + 3, and we need to find the value of f(g(2)).

This can be solved by doing the following steps:

1. Determine g(2), output of the innermost function when input, $x = 2$. $g(2) = 2(2) + 3 = 7$.

2. Now, output of the function g(x), which is 7, becomes the input of the function f(x) or in other words, f(g(2)) becomes f(7).

3. Calculate f(7)

Now, $f(x) = 8x$

So, $f(7) = 8(7)$

$f(7) = 56$

So, $f(g(2)) = 56$

Domain of the function

Domain is the set of all inputs over which the function is defined, or, produces real values.

Range of the function

Range of the set of all the outputs a function can produce.

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Exponents and Radicals

Theory

Exponents are used to simplify the expressions.

A represents A multiplied N times, i.e.

A = A x A x A x A x .... multiplied N times

A is called BASE, and N is called POWER/INDEX/EXPONENT/MULTIPLICITY.

A radical expression is defined as any expression containing a radical (√) symbol. It is used to determine the square root of a number.

Example: $\sqrt{4} = 2$

N^{th} root of A means $\sqrt[N]{A}$ or $A^{1/N}$

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Following are the basic properties and formulae regarding exponents:

Rule name	Rule	Example
Product rules	$a^n \cdot a^m = a^{n+m}$	$2^3 \cdot 2^4 = 2^{3+4} = 128$
	$a^n \cdot b^n = (a \cdot b)^n$	$3^2 \cdot 4^2 = (3 \cdot 4)^2 = 144$
Quotient rules	$a^n / a^m = a^{n-m}$	$2^5 / 2^3 = 2^{5-3} = 4$
	$a^n / b^n = (a / b)^n$	$4^3 / 2^3 = (4/2)^3 = 8$
Power rules	$(b^n)^m = b^{n \cdot m}$	$(2^3)^2 = 2^{3 \cdot 2} = 64$
	$bn^m = b(n^m)$	$2.3^2 = 2(3^2) = 512$
	$^m\sqrt{(b^n)} = b^{n/m}$	$^2\sqrt{(2^6)} = 2^{6/2} = 8$
	$A^{1/n} = {}^n\sqrt{A}$	$8^{1/3} = {}^3\sqrt{8} = 2$

Rule name	Rule	Example
Negative exponents	$b^{-n} = 1 / b^n$	$2^{-3} = 1/2^3 = 0.125$
Zero rules	$b^0 = 1$	$5^0 = 1$
	$0^n = 0$, for $n>0$	$0^5 = 0$
One rules	$b^1 = b$	$5^1 = 5$
	$1^n = 1$	$1^5 = 1$
Minus one rule	$(-1)^n = \begin{cases} 1 & ,n\ even \\ -1 & ,n\ odd \end{cases}$	$(-1)^5 = -1$

Example

$2^x = 16^y$. Find the relation between x and y.

Solution

To equate exponents, first make bases on both sides equal.

$2^x = 16^y \Rightarrow 2^x = 2^{4y}$

$\Rightarrow x = 4y$

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